

Binary and unsigned integers

Lecture B02



Lecture notes section B02

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Last time

- Bits
 - bytes
 - words
- Memory
 - addressing
 - used to store
 - numbers
 - code
 - characters

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In this lecture

- Place value number systems
- Bases
 - binary (base 2)
 - hexadecimal (base 16)
 - octal (base 8)
- Thinking in binary
- Converting
 - from binary
 - to binary
- Unsigned integers

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DICTATA DE MVLTPLICATIONE

CALCVLATE CXC VII A CCXLIII
MVLTPLICATA

$$\begin{array}{r} \text{CXC VII} \\ \times \text{CCXLIII} \\ \hline \text{?????} \end{array}$$

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Place value systems

- Each digit worth a value depending on its position in a number
 - $600 > 60 > 6$
- Ratio between adjacent columns' value is constant
 - 600 is ten times 60
 - 60 is ten times 6
- Ratio is called base of number system
 - most humans use base 10 (decimal)
 - other base values are possible

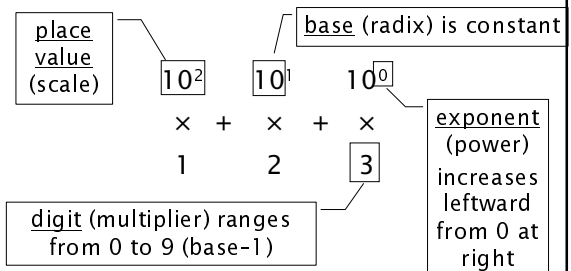
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Place value systems

- What does "123" (decimal) mean?



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Place value systems

- What would "11010" (base 2) look like?

			2		
2 ⁴	2 ³	2 ²	2 ¹	2 ⁰	
×	+	×	+	×	+
1	1	0	1	0	

base is 2

digits are 0 or 1 (because base is 2)

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Place value systems

- What value does "11010" (base 2) have?

2 ⁴	2 ³	2 ²	2 ¹	2 ⁰	
×	×	×	×	×	
1	1	0	1	0	
16	8	0	2	0	= 26

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Place value systems

2	10100111100
3	1211122
4	110330
5	20330
6	10112
7	3623
8	2474
9	1748
10	1340
11	1009
12	938
13	7C1
14	6BA
15	5E5
16	53C

base

Smaller bases: less compact, simpler range of digits

These are all representations of the number 1340 (decimal) in bases from 2 to 16

Higher bases: more compact, wider range of digits

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Place value systems

2	10100111100
3	1211122
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16	53C

base

Digits with value greater than 9 use letters from the alphabet:

A = ten (10)
 B = eleven (11)
 C = twelve (12)
 D = thirteen (13)
 E = fourteen (14)
 F = fifteen (15)
 etc.

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Place value systems

2	10100111100	binary
3	1211122	
4	110330	
5	20330	
6	10112	
7	3623	
8	2474	octal
9	1748	
10	1340	decimal
11	1009	
12	938	
13	7C1	
14	6BA	
15	5E5	
16	53C	hexadecimal ("hex")

base

bases common in computing

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Binary

- In binary (base 2), digits have two values
 - 0 or 1
- A bit can store two different values
- Binary is the natural way to store numbers into bits
 - binary is a less compact representation, but simplicity is more important
- "Bit" stands for "binary digit"

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Thinking binary

- Powers of 2

- useful to memorize up to about 2^{12}

$2^0 = 1$	$2^{12} = 4096$
$2^1 = 2$	$2^{13} = 8192$
$2^2 = 4$	$2^{14} = 16384$
$2^3 = 8$	$2^{15} = 32768$
$2^4 = 16$	$2^{16} = 65536$
$2^5 = 32$	$2^{20} = 1\,048\,576$
$2^6 = 64$	$2^{24} = 16\,777\,216$
$2^7 = 128$	$2^{30} = 1\,073\,741\,824$
$2^8 = 256$	$2^{31} = 2\,147\,483\,648$
$2^9 = 512$	$2^{32} = 4\,294\,967\,296$
$2^{10} = 1024$	
$2^{11} = 2048$	

$2^{10} =$
about
1000
("k")

$2^{20} =$
about
1000000
("M")

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Converting from binary

- Set total to zero
- For each digit (bit):
 - assign it a place value
 - from powers of two list
 - if digit is 1:
 - add place value to total
 - else (digit is 0):
 - do nothing
 - repeat for remaining digits
- Result of addition is value of number

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Converting from binary

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
(128)	(64)	(32)	(16)	(8)	(4)	(2)	(1)	place values
0	1	0	1	1	0	1	0	
x	✓	x	✓	✓	x	✓	x	add?
	64		16	8		2		= 90

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Converting from binary

- Or, in other words . . .

$$01011010_2 = 90_{10}$$

subscripts
sometimes used
to indicate base

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Converting to binary

- Almost all powers of two are even
 - $2^1 = 2$, $2^8 = 256$, $2^{15} = 32768$, ...
 - $2^0 = 1$
- Only final digit contributes to even/odd
 - If number is odd, final digit is 1
 - If number is even, final digit is 0
- To discover final digit, divide by 2
 - remainder (0 or 1) is digit
 - quotient can be used to repeat procedure
 - $2^1/2 = 1$, $2^8/2 = 128$, $2^{15}/2 = 16384$, ...

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Converting to binary

- Start with number to convert
- Repeat:
 - write digit (number % 2)
 - halve number
 - discard remainder
 - stop when number is zero
- Binary number is complete
 - read digits in reverse order of generation

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Converting to binary

	106 ₁₀		
÷ 2 =	53	rem 0	↑ read up
÷ 2 =	26	rem 1	
÷ 2 =	13	rem 0	
÷ 2 =	6	rem 1	
÷ 2 =	3	rem 0	
÷ 2 =	1	rem 1	
÷ 2 =	0	rem 1	

result is 1101010₂

result is zero, stop

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Converting to binary

▪ Or, in other words . . .

right hand bit is sometimes called least significant bit (LSB)

left hand bit is sometimes called most significant bit (MSB)

106₁₀ = 1101010₂

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Thinking binary

binary value	0 = 0000 = 0	
	1 = 0001 = 1	
	2 = 0010 = 2	
	3 = 0011 = 3	
	4 = 0100 = 4	hexadecimal digit
	5 = 0101 = 5	
	6 = 0110 = 6	
	7 = 0111 = 7	
	8 = 1000 = 8	
	9 = 1001 = 9	
decimal value	10 = 1010 = A	
	11 = 1011 = B	
	12 = 1100 = C	
	13 = 1101 = D	
	14 = 1110 = E	
	15 = 1111 = F	

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Hexadecimal

- Base 16
 - digits 0 to 9, A to F
 - often called "hex" for short
- Convenient shorthand for writing binary values
 - easier for humans to read
 - still shows underlying binary representation
 - 16 = 2⁴ (power of two)
- Computers don't use hex internally
 - all memory is bits (binary)

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Binary to hex

fill with zeroes at front to make digit count a multiple of four

010100111100₂

collect binary digits in groups of four digits ("quartets")

write hex equivalent of each binary quartet

53C₁₆

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Hex to binary

5FE1₁₆

write out the binary equivalent of each hex digit

0101111111100001₂

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Octal

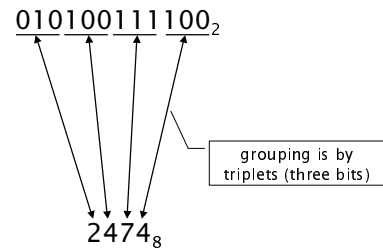
- Base 8
 - digits 0 to 7
- Seldom used, most uses now supplanted by hex
 - but octal still appropriate for some applications, e.g. Unix file permissions

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Octal and binary



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Programming bases in C

- Cannot directly write a literal binary number in C
- Can write a literal hex number by prefixing with 0x (zero, letter x)
 - 0x53c (1340 decimal)
 - a through f can be lower or upper case
- Can write a literal octal number by prefixing with 0 (zero)
 - 02474 (1340 decimal)
- Can write plain decimal number
 - 1340
 - all are stored internally as binary

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Unsigned integers

- One byte holds 8 bits
- What range of binary values can be stored in a byte?
- Smallest value: 00000000
 - all bits are zero, none contribute to number's value
- Largest value: 11111111
 - all bits are one, all contribute to number's value

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Unsigned integers

- Smallest value (8 bits)

2 ⁷	2 ⁶	2 ⁵	2 ⁴	2 ³	2 ²	2 ¹	2 ⁰
(128)	(64)	(32)	(16)	(8)	(4)	(2)	(1)

0 0 0 0 0 0 0 0

x x x x x x x x

= 0

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Unsigned integers

- Largest value (8 bits)

2 ⁷	2 ⁶	2 ⁵	2 ⁴	2 ³	2 ²	2 ¹	2 ⁰
(128)	(64)	(32)	(16)	(8)	(4)	(2)	(1)

1 1 1 1 1 1 1 1

✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓

128+64 + 32 + 16 + 8 + 4 + 2 + 1 = 255

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Unsigned integers

- For 8 bits:
 - smallest representable value is 0
 - largest representable value is 255
 - all numbers in between are representable
 - no other numbers representable
- $255 = 2^8 - 1$

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Unsigned integers

- For n bits
 - smallest representable number is 0
 - all zeroes
 - largest representable number is $2^n - 1$
 - all ones
 - all numbers in between are representable
 - no other numbers are representable

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Unsigned integers

- C has unsigned integers of various sizes
- 8 bits (unsigned char)
 - 0 to 255
- 16 bits (unsigned short)
 - 0 to 65535
- 32 bits (unsigned long)
 - 0 to 4294967295
- 64 bits (unsigned long long)
 - 0 to 18446744073709551615
- `int` is either short or long
 - depends on computer and compiler

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Covered in this lecture

- Place value number systems
- Bases
 - binary (base 2)
 - hexadecimal (base 16)
 - octal (base 8)
- Thinking in binary
 - powers of two
 - binary 0 to 15
- Converting
 - from and to binary
- Unsigned integers

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Going further

- Negative bases
 - base -2 and -10
 - <http://mathworld.wolfram.com/Negabinary.html>
 - <http://mathworld.wolfram.com/Negadecimal.html>
- Balanced ternary
 - base 3 with digit values +1, 0 and -1
 - <http://perun.hscs.wmin.ac.uk/~jra/ternary>
- Fibonacci base system
 - place values are Fibonacci numbers (1, 2, 3, 5, 8, 13, 21, ...)
 - <http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibrep.html>

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Next time

- Binary arithmetic
- Signed binary numbers



Reading:
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