Question 1 [2 + 1 + 2 + 1 + 1 = 7 marks]

Consider the following languages:

1. The set of strings of **a** and **b** which have an even length.
2. **EQUAL.**
3. $\{\mathbf{a^n b^n a^n} : \mathbf{n} \geq \mathbf{0}\}$.
4. **PALINDROME.**
5. **ALAN.**
6. The complement of **ALAN**.
7. **EVEN-EVEN.**
8. $\{\mathbf{a^n b^n} : \mathbf{n} \geq \mathbf{0}\}$.
9. The complement of **EVEN-EVEN**.
10. **EQUAL** $\cap$ $\mathbf{a^* b^*}$.
11. $\mathbf{a^* b^*}$ $\cap$ **EVEN-EVEN.**

For the following criteria state, using the above numbers, which one or more of the above languages fit the criteria.

(i) A context-free language which is not a regular language.

(ii) A computable language which is not a context-free language.

(iii) A regular language.

(iv) A language which is not recursive.

(v) A language whose complement is not recursive.

Note marks will be deducted for any language which is given that does not fit the criteria, but zero is the lowest mark that will be given for any part of a question.

7

Question 2 [1 mark]

Name a problem, given in lectures, which has been proved cannot be solved by a computer.

Question 3 [2 marks]

Give a definition of a Universal Turing Machine.

Question 4 [1 mark]

Describe an application of the Pumping Lemma for Context Free Languages.

Question 5 [3 marks]

Write a regular expression for the complement of the language defined by the regular expression $(\mathbf{ab}+\mathbf{ba})(\mathbf{a}+\mathbf{b})^*$.

Question 6 [4 marks]

Consider the regular expression $-?([\mathbf{0}-\mathbf{9}]^*.[\mathbf{0}-\mathbf{9}]^+)|[\mathbf{0}-\mathbf{9}][\mathbf{0}-\mathbf{9}]^+$

(i) Write two real numbers which are defined by the above regular expression.

(ii) Write two real numbers which are **not** defined by the above regular expression.

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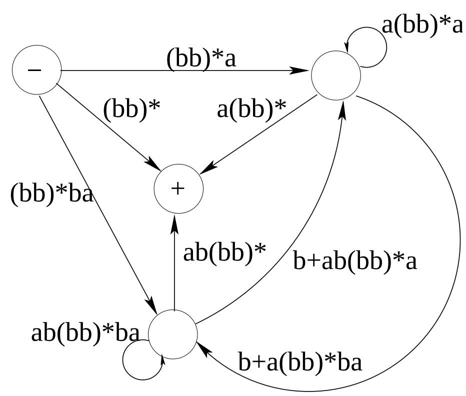
Question 7 [2 + 2 = 4 marks]

(i) Write a regular expression for the language consisting of all strings of **a** and **b** which contain exactly 5 **a**'s.

(ii) Given that an identifier is *a string of letters (where a letter is an alphabetical letter or an underscore _) and digits, and the first character of the identifier is a letter*, write a regular expression defining an identifier.

Question 8 [6 marks]

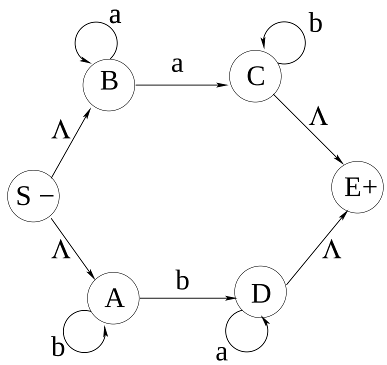
For the following Generalised Transition Graph, show that the following string **abbaabab** is accepted, by stating in order the regular expressions that describe the path from the **Start State** to the **Final State**.



10

Question 9 [4 + 8 + 2 = 14 marks]

(i) Write a Regular Grammar next to the following NFA-Λ.



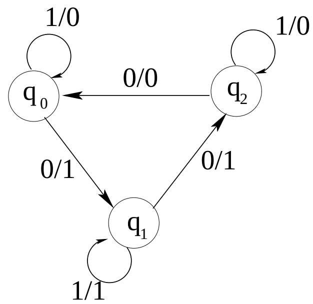
(ii) Convert the above NFA-Λ into a Finite Automaton. **You only need to write down the transition table for the Finite Automaton.**

(iii) Draw the Finite Automaton which corresponds to the complement of the above Finite Automaton.

14

Question 10 [1 + 4 + 1 + 1 + 1 = 8 marks]

(i) For the following Mealy machine write the corresponding output for the input **0011001**.



(ii) Complete the following table for the above Mealy machine.

A	**B**	**Q**	**IN**	**Q***	**A***	**B***	**OUT**
0	0	q_0	0				
		q_0	1				
0	1	q_1	0				
		q_1	1				
1	0	q_2	0				
		q_2	1				

(iii) Write an equation for $\mathbf{A}^*$ in terms of **A**, **B**, and **IN**.

(iv) Write an equation for $\mathbf{B}^*$ in terms of **A**, **B**, and **IN**.

(v) Write an equation for **OUT** in terms of **A**, **B**, and **IN**.

8

Question 11 [2 marks]

Show that the following Context Free Grammar, (where **integer** can be any integer), is ambiguous.

$$S \rightarrow A$$
$$A \rightarrow A + A \mid A * A \mid \textbf{integer}$$

Question 12 [2 marks]

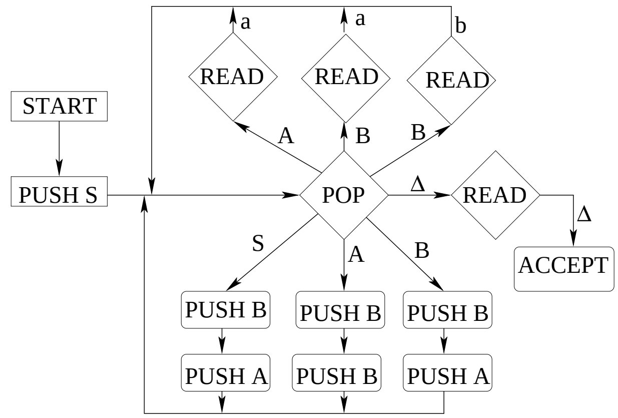
For the following Context Free Grammar, write the left and right derivations for the word $\mathbf{a^*b^* + \Lambda}$

$$S \rightarrow E$$
$$E \rightarrow E + T \mid T$$
$$T \rightarrow TF \mid F$$
$$F \rightarrow F^* \mid (E) \mid a \mid b \mid \Lambda$$

4

Question 13 [6 marks]

Find a path for which the word **bba** is accepted by the following Nondeterministic Pushdown Automaton, and fill in the table below showing what might happen to the **INPUT TAPE** and **STACK** as it proceeds through the machine. Also underline the character currently being read on the **INPUT TAPE** and the symbol on the top of the **STACK**.



STATE	INPUT TAPE	STACK
START	<u>b</u>ba	Δ

6

Question 14 [8 + 2 = 10 marks]

(i) Build a Turing Machine for **EVEN-EVEN** based on the following algorithm.

If the current character is blank then halt.
If the first character is **a**, change it to **X**, else change it to **Y**.
Move up the tape. Check whether the total number of **a**'s and **X**'s are even, and change each **a** to an **A**.
Move down the tape. Check whether the total number of **b**'s and **Y**'s are even, and change each **b** to a **B**.

(ii) Demonstrate that the machine you constructed in **Part (i)** works by tracing its execution on the strings **baaba** and **abab**.

10

Question 15 [9 + 1 = 10 marks]

(i) Build a Turing Machine for the language defined by the following algorithm.

If the current character is **a**, then change it to #.
loop {
  Moving right, find the first **b** and change it to a **B**.
  Moving left, find the first **a** or #, that is to the left of the **b** that was found.
  If an **a** was found, then change it to an **A**. Otherwise change the # to **X**.
  Go back to the start of the tape.
  If the current letter is **X** then {
   Moving right, skip over any **A**'s and **B**'s.
   If the current character is blank, then halt.
   Else if the current character is **a**, then change it to #.
  }
}

(ii) Demonstrate that the machine you constructed in **Part (i)** works by tracing its execution on the string **aabb**.

10

Question 16 [2+1+3 = 6 marks]

Consider the following propositions:

$\mathbf{P_1}$: $\neg \mathbf{P} \vee \mathbf{Q}$

$\mathbf{P_2}$: $\mathbf{Q} \rightarrow \mathbf{R}$

$\mathbf{P_3}$: $\mathbf{P}$

$\mathbf{C}$: $\mathbf{R}$

(i) Use truth tables to show that argument with premises $\mathbf{P_1}$, $\mathbf{P_2}$ and $\mathbf{P_3}$ and conclusion $\mathbf{C}$ is valid.

(ii) Convert each of $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$ and $\mathbf{C}$ to clausal form.

(iii) Give a resolution derivation of the formula $\neg \mathbf{C} \wedge \mathbf{P_1} \wedge \mathbf{P_2} \wedge \mathbf{P_3}$.

6

Question 17 [3 marks]
Indicate which of the following formulas are in Conjunctive Normal Form **(CNF)**, Disjunctive Normal Form **(DNF)**, both **(CNF,DNF)** or neither **(N)** by writing **CNF**, **DNF**, **CNF,DNF** or **N** beside them.

(i) $\mathbf{P} \wedge \mathbf{Q}$

(ii) $\neg\mathbf{R}$

(iii) $(\neg\mathbf{P} \wedge \mathbf{Q}) \vee (\mathbf{Q} \wedge \neg\mathbf{R})$

(iv) $(\mathbf{R} \vee \neg\mathbf{Q}) \wedge (\neg\mathbf{P} \vee \neg\mathbf{R})$

(v) $\neg(\mathbf{P} \wedge \mathbf{Q}) \vee \mathbf{R}$

(vi) $\neg\mathbf{P} \vee \neg\mathbf{Q}$

Question 18 [7 marks] Translate the following into formulas of first order logic using the dictionary given.

dog(X)	for	**X is a dog.**
cat(X)	for	**X is a cat.**
run(X)	for	**X run(s).**
chases(X,Y)	for	**X chases Y**

(i) Spot is a dog.

(ii) Tiger is a cat who runs.

(iii) Cats never chase dogs.

(iv) Some cats run if chased by a dog.

(v) All dogs chase some cats.

10

Question 19 [4 marks]

Using resolution show that the following set of clauses are unsatisfiable. Clearly indicate at each step which clauses you are resolving and what unifications you are making.

$$\{\{\mathbf{F(c)}\}, \{\neg\mathbf{G(Y)}, \mathbf{H(c, Y)}\}, \{\neg\mathbf{F(X)}, \neg\mathbf{B(Y)}, \neg\mathbf{H(X, Y)}\}, \{\mathbf{G(a)}\}, \{\mathbf{B(a)}\}\}$$

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