Bayesian Networks

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Overview

- Random Variables
- Terminology
- Bayesian (Belief) Networks
- Inference in Bayesian Networks
- Arc Reversal

Random Variables

- *Random Variable*
  - A function that assigns to each outcome a value.
- Boolean Random Variable
  - A random variable which has only the two values **{true, false}**.
  - Associated with propositional statements.
- Discrete
  - A random variable which has only a finite or countable number of values.
- Continuous
  - A random variable which can have a set of values containing an interval of real numbers.

Holmes Burglary Alarm (Pearl)

- Holmes lives in an area where there is a 1% chance your house will be burgled.
- Holmes' alarm will sound 94% of the time he is burgled, and 0.1% of the time for no apparent reason.
- The Baker Street Irregulars (Kids in the neighbourhood) will call 90% of the time the alarm sounds and 5% of time when the alarm does not sound for other reasons.
- A friend Watson will call 70% of the time when the alarm sounds, and 10% of the time when the alarm does not sound.

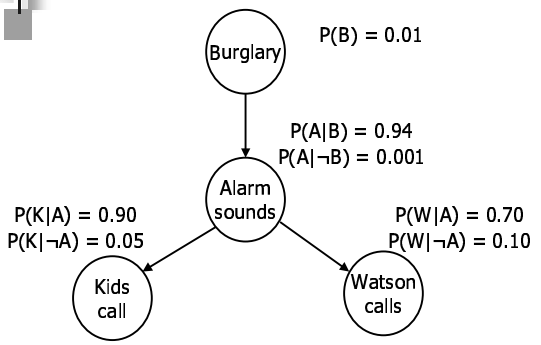
Was Holmes burgled?

- Holmes is told that Watson has called. What is the probability that Holmes' house was burgled?
- Suppose that Holmes is now told that the Baker Street Irregulars have also called. What is now the probability that Holmes' house was burgled?

Joint Probability

Burglary	Alarm	Kids	Watson	*Probability*
T	T	T	T	*.005922*
T	T	T	F	*.002538*
T	T	F	T	*.000094*
T	T	F	F	*.000846*
T	F	T	T	*.000021*
T	F	T	F	*.000009*
T	F	F	T	*.000057*
T	F	F	F	*.000513*
F	T	T	T	*.000624*
F	T	T	F	*.000267*
F	T	F	T	*.000010*
F	T	F	F	*.000089*
F	F	T	T	*.034615*
F	F	T	F	*.014835*
F	F	F	T	*.093956*
F	F	F	F	*.845604*

Holmes' Alarm Network



Joint Probability

P(B = b, A = a, K = k, W =w)
= P(W= w|A = a)P(K = k|A = a)P(A = a|B = b)P(B = b)

Therefore:

P(B = b, W=w)
= P(W=w|A=*True*)P(A=*True*)|B=b)P(B=b)
+ P(W=w|A=*False*)P(A=*False*)|B=b)P(B=b)

Bayesian Network

- Data Structure which represents the dependence between variables.
- Gives a concise specification of the joint probability distribution.
- A Bayesian Network is a DAG such that:
  - Nodes represent random variables.
  - Associated with every node is the condition probability of the node given its parents.

AIDS Example

- Suppose 0.6% of the population have AIDS.
- Suppose we have a test for AIDS such that:
  - For a person with AIDS the test would give a positive result with probability 0.977
  - For a person without AIDS the test would give a negative result with probability 0.926

*What would be the probability that if the result of the test was positive for a person, then that person did **not** have AIDS?*

Urn Example

- Two urns. First one has 70 green balls and 30 red balls. The second has 70 red balls and 30 green balls.
- A urn is chosen at random and you draw 12 balls with replacement, 8 green balls and 4 red balls.

What is the probability that the urn chosen is the one with 70 green balls and 30 red balls*?*

(Example from Raiffa, 1968)

Stud Farm

The stallion Brian has sired Dorothy with the mare Ann and sired Eric with the mare Cecily. Dorothy and Fred are the parents of Henry, and Eric has sired Irene with Gwenn. Ann is the mother of both Fred and Gwenn, but their fathers are in no way related. The colt John, whose parents are Henry and Irene, is born with a rare hereditary disease carried by a recessive gene. What are the probabilities for the remaining horses to be carriers of the gene responsible for the disease?

(Example from Jensen, 1996)

Asia

A patient comes to a doctor with shortness of breath. The doctor considers that the possible causes are tuberculosis, lung cancer and bronchitis. Other additional information that is relevant is whether the patient has recently visited Asia (where tuberculusis is more prevalent), whether or not the patient is a smoker (which increases the chance of cancer and bronchitis). A positive xray would indicate either tuberculosis or lung cancer.

(Example from Lauritzen and Spielgelhalter 1988).

Inference in Bayesian Networks

- Compute the posterior probability distribution of a set of query variables, given values for some evidence variables.
- Also called Belief updating.
- Types of inference:
  - Diagnostic
  - Causal
  - Intercausal
  - Mixed

Types of Inference