School of Computer Science and Software Engineering
Monash University

Bachelor of Arts/Bachelor of Science (Honours) (1620)
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Information Theoretic Image Thresholding

by

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1 Introduction

Image thresholding uses an image's histogram in order to sub-divide an image into non-overlapping regions, by classifying the image's pixels according to their values. The simplest case for thresholding is a greyscale image consisting of two distinct regions, i.e., an object region and a background region. The pixels whose grey-level values are less than or equal to the threshold are assigned to one segment and the pixels whose grey-level values are greater than the threshold are assigned to another segment. If a good threshold value is chosen, the object region should be clearly distinguishable from the background region in the resulting binary image.

Mixture modelling (Wallace and Boulton, 1968) is the process of finding classes of 'things' with similar attributes that best represent the given data. In the case of image thresholding, this would be the classification of the image's pixels into two component probability distributions that best approximate the image's histogram.

Using a mixture modelling approach to thresholding, a measure is needed to indicate how well a histogram is approximated by a particular mixture model. The Directed Divergence (the symmetric form of the Kullback-Leibler function) was shown in an investigation of the relative entropy thresholding technique (Bertolo, 2001) to be one of the best ways to measure the discrepancy between the component probability distributions and an image's histogram. This leads to the idea that such a measure could be used as an objective criterion to indicate how well a mixture model approximates the histogram.

The focus of Bertolo's thesis was on improving relative entropy thresholding and as such did not take into account the complexity of the mixture models when calculating the discrepancy between the two distributions. The Minimum Message Length (MML) principle (Wallace and Boulton, 1968) can take this complexity into account by utilising both the mixture model and the Kullback-Leibler measure in a two-part MML message.

1.1 Aims

One aim of this project is to extend the work completed by Bertolo and investigate thoroughly thresholding as an approximation to an image's histogram within the mixture modelling framework. Since mixture modelling generalises gracefully to mixtures of more than two components and to multi-dimensional components, how this approach extends to images requiring multiple thresholds and to multi-spectral images will also be investigated. Another aim is to examine the relative and absolute merits of the Kullback-Leibler measure and its possible use as an objective criterion for the qualitive assessment of thresholding results for an image, and to extend this criterion to the MML principle.

2 Research Context

Within the mixture modelling framework thresholding can be viewed as an approximation of an image's histogram by a mixture of two component probability distributions; the threshold being the value for which both component probability distributions are equally likely. The best threshold would be found when the component probability distributions best approximated the image's actual histogram. Mixture modelling would allow a consideration of arbitrary probability distributions for the components of the mixture model, and would allow generalisation to multi-dimensional histograms.

The work undertaken by Bertolo (2001) focused on modifying relative entropy thresholding proposed by Chang et. al. (1994). This technique thresholds an image by minimising the discrepancy between an ideal approximation of an image's histogram and the image's actual grey-level probability distribution. Bertolo noted that the relative entropy technique approximates the image's histogram with two uniform probability distributions, which, in most cases, is not likely to be an accurate approximation of a natural image's actual histogram. As a solution to this, he modified the relative entropy thresholding approach by substituting Gaussian and Poisson distributions for the components of the ideal distribution. This adaptation produced consistently good results compared to other entropic techniques and the results can be viewed as specific cases of thresholding as image histogram approximation by mixture modelling.

For the task of image thresholding, an objective criterion to evaluate the appropriateness of a mixture model and the resulting threshold may prove valuable. Bertolo continued work left unfinished by Ray et. al. (1997). on probabilistic distance criteria. He experimented with various probabilistic distance measures to best calculate the discrepancy between the two probability distributions for relative entropy thresholding. The Kullback-Leibler measure and the Bhattacharya measure performed the best, but it is the Kullback-Leibler measure that is of interest for this project as it relates directly to MML.

According to the MML principle (Wallace and Boulton 1968), the best encoding of the given data is the one that produces the shortest two-part message. The first part of the message describes a model that represents the data, and the second part of the message is the data encoded used the model described in the first part. The length of the second part of the message is the Kullback-Leibler measure. For image thresholding, the first part of an MML message would describe the mixture model and the second part would encode the histogram using the mixture model. Using MML two-part messages can be seen as extending Bertolo's approach to take the complexity of the mixture model into account.

An alternative to Bertolo's approach for approximating the histogram is to use the classification program *Snob* (Wallace and Dowe, 1997) to fit the best mixture model. *Snob* uses the MML principle from inductive inference to classify data in a way that produces the shortest message length, hence a strength of *Snob* is that it takes into account the complexity of the components when approximating the best mixture model for the given data.

Preliminary results obtained from applying MML to the task of thresholding (Tischer et. al. 1995) suggest that the best or most meaningful threshold values for a limited number of synthetic and natural images were those whose two-part messages were the shortest.

3 Research Plan and Methods

3.1 Methodologies

The examination of thresholding as an approximation of an image histogram will lead to several experiments involving both synthetic and natural images. Synthetic images will be used with specific probability distributions as well as the usual test set of natural images. Initially a modelling approach as used by Bertolo (2001) to fit a mixture model to a histogram will be used, and the Kullback-Leibler measure calculated for different mixture models using combinations of arbitrary components to approximate the image's histogram. The results will be evaluated to assess the Kullback-Leibler measure.

Snob will next be used to preprocess the histograms of the same images to find the appropriate mixture model(s) and the message length, which will then be compared to the above results.

Following on from this, the mixture modelling approach will then be generalised to colour and multi-spectral images in a similar manner.

3.2 Proposed Thesis Chapter Headings

The proposed thesis chapter headings are as follows;

1. Introduction and Aims
2. Definitions
3. Mixture Modelling
   - 3.1 Fitting the Component Mixtures
   - 3.2 Generalisation to Multiple Thresholds
   - 3.3 Generalisation to Multiple Dimensions
   - 3.4 Mixture Modelling using Snob
4. The Kullback-Leibler Measure
   - 4.1 Relative Measure
   - 4.2 Absolute Measure
5. Minimum Message Length Inductive Inference
   - 5.1 MML and Image Thresholding
6. Results
7. Analysis of Results
8. Conclusion
9. Future Work
10. Bibliography
11. Appendix A: Images
12. Appendix B: Code

3.3 Timetable

The following timetable provides a general outline of the activities involved in undertaking this research.

Week/Semester	**Activity**	**Date**
8/I	Research Proposal Finalised	2/05/02
9/I	Commence Literature Review	6/05/02
10/I	Commence testing	13/05/02
11/I	Thesis Chapter 2: Definitions	20/05/02
12/I	Prepare for Interim Presentation	27/05/02
13/I	Interim Presentation	6/06/02
13/I	Prepare Literature Review Draft	7/06/02
1/mid-year break	Literature Review Draft Finalised	13/06/02
2/mid-year break	Thesis Chapter 3: Mixture Modelling	17/06/02
4/mid-year break	Thesis Chapter 4: KL Measure	1/07/02
5/mid-year break	Thesis Chapter 5: MML	8/07/02
2/II	Literature Review Finalised	1/08/02
4/II	Thesis Draft Finalised	12/09/02
5/II	Finish Testing	19/08/02
6/II	Thesis Chapter 7: Analysis of Results	26/08/02
12/II	Prepare for Final Presentation	14/10/02
1/end year break	Final Presentation	28/10/02
1/end year break	Research Logbook to Supervisors	1/11/02
2/end year break	Submit Final Thesis	5/11/02

3.4 Special Facilities Required

All facilities required to complete this project are available at the Clayton Campus, Monash University.

4 Relevance of the Research

The task of thresholding can be subjective in in both the selection of the thresholding technique to be used and in the assessment of the results obtained. More often than not the easiest way to decide if an image has been segmented meaningfully is to simply look at the image. This research aims to provide an objective criterion for the assessment of thresholding results.

5 References

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