

School of Computer Science & Engineering —
UNSW

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An Introduction to the B Method

A Simple Library

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To simplify the model we will assume that there is a set **BOOK** that contains all the books that could be in the library. This could be thought of as similar to the set of all **ISBN** numbers, but as there will be at most only one copy of any book in the library it's more appropriate to compare it with a shelf number, or a book barcode. When you borrow a book from a library, you don't borrow a book title, you borrow a specific physical book.

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The specification we are going to develop is probably not wrong when compared with a real library, nor inappropriate, rather it is **incomplete**.

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Thus the machine header is

MACHINE

SimpleLibrary(BOOK,maxuser)

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CONSTRAINTS          maxuser : NAT1
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We wish the set `USER` to have exactly `maxuser` elements, and we use the `PROPERTIES` clause to constrain the cardinality of the set.

card(USER) = maxuser

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books in the library: the set of books acquired by the library.

books on the shelf: the subset of the library books that are currently on the shelves,
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books on loan: information on what books have been borrowed and who has borrowed them.

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Note: $users \subseteq USER$ is equivalent to $users \in \mathbb{P}(USERS)$.

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A borrower may borrow more than one book.

This indicates a functional relation between `books` and `borrowers`.

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Both of these can be expressed by saying that *books_on_shelf* must be exactly the difference between *books_in_library* and the **domain** of the *books_on_loan* function .

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Thus we need the following constraint

$$books_on_shelf = books_in_library - dom(books_on_loan)$$

Thus we obtain the following header for the **SimpleLibrary** machine.

```
MACHINE          SimpleLibrary(BOOK,maxuser)
CONSTRAINTS     maxuser : NAT1
SETS            USER
PROPERTIES      card(USER) = maxuser
VARIABLES
  users, books_in_library, books_on_shelf, books_on_loan
INVARIANT
  users <: USER &
  books_in_library <: BOOK &
  books_on_shelf <: books_in_library &
  books_on_loan : books_in_library +-> users &
  books_on_shelf = books_in_library - dom(books_on_loan)
```

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INITIALISATION

```
users,  
books_in_library,  
books_on_shelf,  
books_on_loan := {}, {}, {}, {}
```

Adding a Book to the Library

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We want to model an operation `AddBook(book)` that adds book to the libraries collection, the set `books_in_library`.

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`book` must be a new book—one that is not already contained in the libraries collection—to the library collection.

We will assume that at the same time as we add the book to the library collection we add it to the library shelves.

```
AddBook(book) =
```

```
  PRE book : BOOK & book /: books_in_library
```

```
  THEN books_in_library := books_in_library \/ {book} ||
```

```
       books_on_shelf := books_on_shelf \/ {book}
```

```
END
```

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This operation will return the user token that must be used when borrowing a book.

In modelling this operation we choose any user token that has not yet been allocated. We then add this to the set `user` and return the value to the invoker of the operation.

```
newuser <-- NewUser =  
  PRE users /= USER  
  THEN  
    ANY user  
    WHERE user : (USER - users)  
    THEN users := users \/ {user} ||  
      newuser := user  
    END  
  END  
END
```

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Notice that we replace *books_on_loan* by the union of two functions, and this must be a function. In general, the union of two functions is not a function. Why?

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Why is it in this case?

```
Borrow(user,book) =
```

```
  PRE user : users & book : books_on_shelf
```

```
  THEN books_on_shelf := books_on_shelf - {book} ||
```

```
       books_on_loan := books_on_loan \/ {book |-> user}
```

```
END
```

Returning a Book

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Note the use of `domain subtraction` to remove all maplets $book \mapsto anyone$ that records the borrowing of the book by *anyone*. Since this is a function there will be at most one such maplet. In this case there will be exactly one. Why?

```
Return(book) =
```

```
  PRE book : dom(books_on_loan)
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  THEN books_on_shelf := books_on_shelf \ / {book} ||
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       books_on_loan := {book} << | books_on_loan
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An enquiry operation is an operation that does not change the state of the machine.

```
user <-- Borrowed(book) =  
  PRE book : dom(books_on_loan)  
  THEN user := books_on_loan(book)  
  END
```

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$$x \in S, x \subseteq S, x \subset S, \text{ or } x = E \text{ where } x \setminus S \text{ and } x \setminus E^a$$

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Consider the *book* argument to any operation.

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We cannot write simply $book \notin books_on_shelf$, we must write

$book \in BOOK \wedge book \notin books_on_shelf$.

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In the case of the SimpleLibrary machine there is one undischarged Context proof obligation

$$cst(\text{SimpleLibrary}) \Rightarrow \exists \text{USER}. (\text{card}(\text{USER}) = \text{maxuser} \wedge \text{card}(\text{USER}) \in \mathbb{N}_1)$$

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In the case of the `SimpleLibrary` machine there is one undischarged `Context` proof obligation

$cst(\text{SimpleLibrary}) \Rightarrow \exists \text{USER}. (\text{card}(\text{USER}) = \text{maxuser} \wedge \text{card}(\text{USER}) \in \mathbb{N}_1)$

A very simple rewrite rule, $(P \ \& \ Q) == (Q \ \& \ P)$ leads to a proof!

Animation

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Try animating the [SimpleLibrary](#) machine.

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Don't bother instantiating any of the deferred sets.

Populate the books and users of the library by using symbolic names such as `BigBlueBook`, `LittleRedBook` for books and `john`, `jill` for users. All deferred sets are in reality sets of natural numbers and the names suggested above are natural number constants.

books_on_shelf: **A Dependent Variable**

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Given

$$books_in_library \subseteq BOOK \wedge$$

$$books_on_loan \in books_in_library \rightarrow users \wedge$$

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This shows that *books_on_shelf* is a **dependent** variable.

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$$books_on_shelf \subseteq books_in_library$$

This shows that *books_on_shelf* is a **dependent** variable.

There is nothing wrong with having a dependent variable, but we could remove *books_on_shelf* as a variable, and leave the concept of *books_on_shelf* by inserting a **definitions** clause:

$$books_on_shelf \hat{=} books_in_library - \text{dom}(books_on_loan)$$

`books_on_shelf == books_in_library - dom(books_on_loan)` in
ASCII.

Fragile and Robust Operations

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It should be observed that developments conducted completely within the B Method will entail proving that all preconditions are satisfied.

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The following slides show an API version of **SimpleLibrary**.

MACHINE SimpleLibraryAPI (BOOK , maxuser)

CONSTRAINTS maxuser $\in \mathbb{N}_1$

INCLUDES SimpleLibrary (BOOK , maxuser)

SETS

RESPONSE = { OK ,
BookInLibrary ,
NoNewUsers ,
NotRegisteredUser ,
BookNotForLoan ,
BookNotOnLoan }

OPERATIONS

```
response ← AddBookR ( book ) ≐
PRE  book ∈ BOOK
THEN IF book ∉ books_in_library THEN
    AddBook ( book ) ||
    response := OK
ELSE response := BookInLibrary
END
END ;
```

response , newuser \leftarrow NewUserR $\hat{=}$

IF users \neq USER

THEN newuser \leftarrow NewUser ||

 response := OK

ELSE newuser \in USER ||

 response := NoNewUsers

END ;

response \longleftarrow BorrowR (user , book) $\hat{=}$

PRE user \in USER \wedge book \in BOOK

THEN

SELECT

user \notin users THEN response := NotRegisteredUser

WHEN

book \notin books_on_shelf THEN response := BookNotForLoan

ELSE

Borrow (user , book) ||

response := OK

END

END ;

response \longleftarrow ReturnR (book) $\hat{=}$

PRE book \in BOOK

THEN IF book \in dom (books_on_loan) THEN

 Return (book) \parallel

 response := OK

ELSE response := BookNotOnLoan

END

END ;

```
response , user ← BorrowedR ( book ) ≐
PRE  book ∈ BOOK
THEN IF  book ∈ dom ( books_on_loan ) THEN
    response := OK ||
    user ← Borrowed ( book )
ELSE  response := BookNotOnLoan ||
    user ∈ USER
END
END
END
```

Use of SELECT Substitution

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This achieves `non-deterministic` choice in the case that both guards `user \notin users` and `book \notin books_on_shelf` are true, ie a person who is not a registered user is attempting to borrow a book that is not available for loan.

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The specification says that either `NotRegisteredUser` or `BookNotForLoan` are valid responses and we don't care which is chosen.

Enumerated Sets

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Enumerated sets are sets of natural numbers with the symbolic values being mapped onto 0, 1 etc.

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Thus we have `partial hiding` of the state.

Notice that when a machine operation is used, the syntax is the same as that used for the specification of the operation. See, for example, the use of *Borrowed* within *BorrowedR*.

Non-deterministic Choice from a Set

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ASCII $x :: S$

Publication $x \in S$

Arbitrarily choose a value from the set S , and substitute in the variable v .

This substitution is used in `NewUserR`, the robust version of the operation `NewUser`, in the event that it is not possible to choose a new user token.

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When the proof obligations for **SimpleLibraryAPI** are generated it will be noted that there are no proof obligations for the operations.

This is a consequence of the guards on the **IF THEN ELSE** substitutions satisfying the preconditions of the referenced fragile operations from **SimpleLibrary**.

If you wish you can reset the machine, and choose **generate all proof obligations** in the **Options/Provers** menu, and then regenerate the proof obligations. You will now get proof obligations for the operations. They are trivial, but display the proof obligations thrown up by the preconditions of the fragile operations.