

MACHINE Q3R

As for Q2R, but it is known that s is ordered, ie $i \leq j \Rightarrow s(i) \leq s(j)$

REFINES Q3

SEES Q3_ctx

VARIABLES

pos

sx sd is s extended at each end to remove problems with x being $\leq s(1)$ or $\geq s(n)$

lo

hi

INVARIANTS

inv1 : $sx \in 0 .. n + 1 \rightarrow X$

inv2 : $\forall p \cdot p \in 1 .. n \Rightarrow sx(p) = s(p)$

inv3 : $sx(0) \leq s(1)$

inv4 : $sx(0) \leq x$

inv5 : $sx(n + 1) > s(n)$

inv6 : $sx(n + 1) > x$

inv7 : $lo \in 0 .. n$

inv8 : $hi \in 1 .. n + 1$

inv9 : $lo + 1 \leq hi$

inv10 : $sx(lo) \leq x \wedge x < sx(hi)$

THEOREMS

thm1 : $lo \leq (lo + hi)/2$

thm2 : $(lo + hi)/2 < hi$

thm3 : $lo + 1 \neq hi \Rightarrow lo < (lo + hi)/2$

EVENTS

Initialisation

begin

act1 : $pos := 0$

act2 : $sx : |sx' \in 0 .. n + 1 \rightarrow X \wedge$
 $(\forall p \cdot p \in 1 .. n \Rightarrow sx'(p) = s(p)) \wedge$
 $sx'(0) \leq s(1) \wedge sx'(0) \leq x \wedge$
 $s(n) < sx'(n + 1) \wedge x < sx'(n + 1)$

act3 : $lo := 0$

act4 : $hi := n + 1$

end

Event Search $\hat{=}$

Refines Search

when

grd1 : $lo + 1 = hi$

$grd2 : lo \neq 0$
 $grd3 : s(lo) = x$

with

$p : p = lo$

then

$act1 : pos := lo$

end

Event SearchHi $\hat{=}$

Which is convergent

any

mid

where

$grd1 : lo + 1 \neq hi$

$grd2 : mid = (lo + hi)/2$

$grd3 : x < sx(mid)$

then

$act1 : hi := mid$

end

Event SearchLo $\hat{=}$

Which is convergent

any

mid

where

$grd1 : lo + 1 \neq hi$

$grd2 : mid = (lo + hi)/2$

$grd3 : sx(mid) \leq x$

then

$act1 : lo := mid$

end

VARIANT

$hi - lo$

END