Outline

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1 Objectives

Objectives for this Module

1 an understanding of the Fundamentals of the Event-B Method

3 A reading knowledge of Event-B specifications

9 Skills in using the Event-B notation to develop and prove software specifications

12 The ability to write basic Event-B specifications

2 Defining Sets

Set Theory

- Sets are *unordered collections* of *elements*
- Elements are usually named with lower case letters
- Sets are usually named with capital letters
- Concept of *set membership*
- Example: $barina \in HOLDENS$

Defining Sets

- Two basic ways:
  - enumeration
  - comprehension
- Enumeration: $\{barina, astra, commodore\}$
- Comprehension: $\{x |\ x \in S \wedge P\}$
  - S defines the type of x
  - P is a predicate in x, constraining the set
- Example: $\{x |\ x \in \mathbb{N} \wedge x \leq 10\}$
  - read *(the set of natural numbers) x such that x at most 10*
  - note a) typing, and b) constraint

3 Operations on Sets

New Sets from Old

- Union: $S \cup R$
- Intersection: $S \cap R$
- Powerset: $\mathbb{P}(S)$
- Cartesian Product: $S \times R$

See the B summary for formal definitions of these

4 Relations

Relations

- A *relation* $S \leftrightarrow R$ is a formal statement of the correspondence between elements of two sets
- Example: $CARS \leftrightarrow PRICE$
- A relation is a set of sets of ordered pairs: $S \leftrightarrow R = \mathbb{P}(S \times R)$
- Note the effect of the powerset: the set of relations $S \leftrightarrow R$ includes the empty set (no relation), the set of all ordered pairs $(S \times R)$ (everything related to everything), as well as all subsets in between.
- the set of elements in the left of the ordered pairs is called the *domain*
- the set of elements in the right of the ordered pairs is called the *range*

Relation Examples

- No price information known: $\{\}$
- *barina* costs \$13990: $\{(barina, 13990)\}$
- *astra* costs \$25490: $\{(barina, 13990), (astra, 25490\}$
- "optioned up" *barina* costs \$25490: $\{(barina, 13990), (barina, 25490), (astra, 25490)\}$
- Note that elements in both domain and range need not be unique.
- $dom(BULLET4) = \{barina, astra\}$
- $ran(BULLET4) = \{13990, 25490\}$

5 Functions

Functions

- special case of relations: elements in domain are unique
- not all elements need be in domain: *partial*
- further special cases:
  - all elements in domain: *total*
  - elements in range unique: *injective*
  - all elements in range: *surjective*
- injective functions are also called *one-to-one*
- surjective functions are also called *onto*
- injective/surjective functions can be partial or total
- a function that is both injective and surjective is called *bijective*

Use of Functions and Relations for Data Modelling

- Since B does not have any data structures in the conventional sense, functions and relations are used
- For example, use function to model an array
- $array := \{(0, 45), (1, 23), (2, 16), (3, 18)\}$
- $baseprice := \{x \mapsto y |\ x \in HOLDENS \wedge y \in \mathbb{N}\}$
- $x \mapsto y$ is called a *maplet* (ordered pair)
- since the result of a function can itself be a function, can *use functions to model classes!*

6 Summary

Summary

- *Sets* are an essential component of the B Experience
- understand *Relations and Functions* as sets
- use Relations and Sets as *data structures and class modelling tools*