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Machine Plotting of Radio/Radar Vertical-Plane Coverage Diagrams

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Please make the following corrections in Appendix B:

1. On page 44, change lines 23 and 29, which read

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FF = 10.**(FDB*.1)
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to

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FF = 10.**(FDB*.05)
```

2. On page 46, between the statements "F2 = F1" and "GO TO 4" insert a new statement reading

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FN = F1
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CONTENTS

ABSTRACT

Fortran computer subroutines have been developed for machine plotting of radar or radio vertical-plane coverage diagrams for the case of interference between direct and sea-reflected waves. One set of subroutines plots detection or constant-signal-level contours on a range-height-angle chart. A second set plots signal level in decibels as a function of range for a target at a fixed (low) altitude. The first type of plot is valid for antenna heights that are within a few hundred feet of the water and for targets that are at much higher altitudes and not too close to the horizon. The second type is valid for antenna and target heights that are both less than a few thousand feet altitude, with no restriction on minimum altitude. Normal atmospheric refraction is taken into account, but scattering and ducting are assumed to be of negligible effect. The frequency range considered is from about 100 MHz to 10 GHz. The factors taken into account are frequency, antenna and target heights, antenna beamwidth, tilt of the beam with respect to the horizon, roughness of the sea (wave height), polarization of the radio waves, and the calculated or assumed free-space range. Sample plots are shown and discussed. The plotting techniques employed are briefly described, and listings of the essential Fortran subroutines are given in an appendix.

PROBLEM STATUS

This is an interim report on a continuing NRL Problem.

AUTHORIZATION

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MACHINE PLOTTING OF RADIO/RADAR VERTICAL-PLANE COVERAGE DIAGRAMS

INTRODUCTION

When the antenna of a radar or radio station in the VHF to microwave frequency range overlooks a reflecting surface, such as the sea, the free-space radiation and reception patterns are modified by the presence of a reflected wave, in addition to the wave that goes by a direct path from the transmitter to the receiver (by way of the target in the radar case). The modified pattern in the vertical plane is the result of phasor-vector addition of the direct and reflected waves; it is an interference pattern, with alternate maxima and minima, or lobes and nulls. In optics this phenomenon is known as the Lloyd's mirror effect. It is an effect well known to naval radar operators, and the mathematics of the problem have long been well understood. In the lobe maxima, the radar echo signal may be increased by as much as 12 dB compared to the free-space signal. In the minima, or nulls, on the other hand, the signal strength may theoretically go to zero.

It is frequently desired to make a plot of the vertical-plane coverage diagram of a radar, taking into account this interference effect, in order to predict regions in which targets will and will not be detected. The computations required for such a plot, and the actual plotting, are extremely tedious even though basically straightforward. The obvious present-day solution of this type of problem is the use of digital machine computation and machine plotting. The purpose of this report is to describe a procedure of this type that has been developed, and to show some typical resulting plots. It is believed that this procedure constitutes a valuable tool for the system designer and operational analyst. It allows detailed plots to be produced in a short time and at moderate cost. These plots enable the effects of various parameters to be studied visually. To attempt the same thing by manual methods would take an inordinately long time and would be prohibitively expensive.

INTERFERENCE OF DIRECT AND REFLECTED WAVES

Pattern-Propagation Factor

The geometry of the interference problem is shown in Fig. 1, with heights greatly exaggerated relative to range. The mathematical formulation of the problem that follows is a modification and slight generalization of the one given by Kerr (1), except for Eqs. (24), (25), and (26), which are derived in Appendix A.

It is apparent that the direct-path wave and the reflected wave traverse different distances in going from the antenna to the target; their path difference is

$$\delta = R_1 + R_2 - R . \tag{1}$$

The phase difference of the two interfering waves due to this path difference is $2\pi\delta/\lambda$ radians, where λ is the radar wavelength. The total phase difference α of the two waves is this path-length phase difference plus the phase change ϕ that occurs in the reflection process.

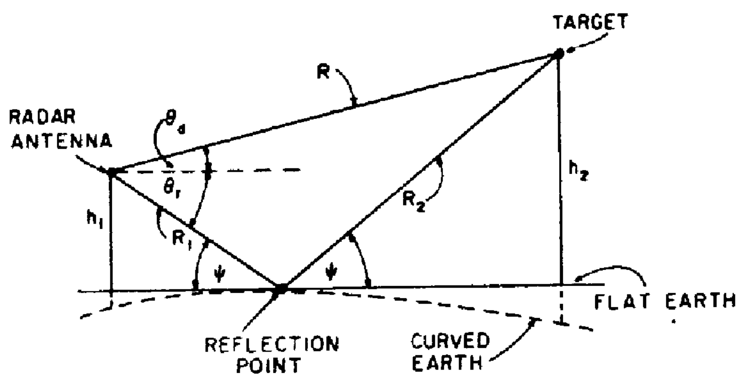


Fig. 1 - Geometry of the surface-reflection interference problem

If the two interfering waves have (at least approximately) the same vector (spatial) direction, then the most important factor in their addition is their phasor relationship. The resulting electric-field phasor is

$$E = |E_d| + |E_r| \; e^{-j\alpha} , \tag{2}$$

where E_d is the electric field of the direct-path wave, E_r is that of the reflected wave, and α is their phase difference.

Kerr (1) defines the propagation factor F as the ratio of the actual field $|E|$ at a point in space to that which would exist in free space at the same distance, $|E_0|$. If $|E_d|$ is assumed equal to $|E_0|$, as it will be in the absence of absorption losses or abnormal refractive effects, from Eq. (2) it is apparent that

$$F = \frac{|E|}{|E_0|} = \left| 1 + \frac{|E_r|}{|E_0|} \; e^{-j\alpha} \right| . \tag{3}$$

If the path difference δ is assumed to be very small compared to R (as will always be true in situations of practical interest), for a flat smooth surface the ratio $|E_r|/|E_0|$ is equal to the magnitude ρ_0 of the reflection coefficient of the reflecting surface. Therefore

$$F = |1 + \rho_0 \, e^{-j\alpha}| = |\sqrt{1 + \rho_0^2 + 2\rho_0 \cos \alpha}| . \tag{4}$$

If the surface is not smooth and flat, ρ_0 is replaced by ρ', which is the product of three separable factors: a factor dependent on the roughness of the surface, one that depends on its curvature (e.g., sphericity of the earth), and one that expresses the inherent reflectivity of the material. If the reflection coefficient is ρ_0 for a plane smooth surface, a roughness factor r and a divergence factor D can be defined to take into account, respectively, the roughness and the curvature. Then

$$\rho' = rD\rho_0 . \tag{5}$$

Thus far F has been defined as simply the "propagation factor." It is necessary, however, to take into account in this factor that the antenna vertical-plane pattern can also cause a difference in strength of the direct and reflected rays. The factor $f(\theta)$ defines the antenna pattern. It is the ratio of the field strength in direction θ relative to that in the beam-maximum direction, where θ is angle measured with respect to an arbitrary reference direction in the vertical plane. If θ_1 (Fig. 1) is the direction of the direct ray as it leaves the antenna and θ_2 is the direction of the reflected ray at the same point, it may or may not be true that $f(\theta_1) = f(\theta_2)$. It is therefore necessary to define the *pattern*-propagation factor (which F will henceforth denote) as

$$F = |f(\theta_1) + \rho' f(\theta_2)\, e^{-j\alpha}|$$

$$= f(\theta_1) \left| 1 + \rho' \frac{f(\theta_2)}{f(\theta_1)} e^{-j\alpha} \right| . \tag{6}$$

As thus defined, F is the ratio of the actual field strength at a point in space to that which would exist at the same distance from the antenna in free space *in the beam maximum.*

A generalized reflection coefficient, x, can now be defined:

$$x = \rho' \frac{f(\theta_2)}{f(\theta_1)} = \frac{rD\rho_0 f(\theta_2)}{f(\theta_1)} . \tag{7}$$

The pattern propagation factor then becomes simply

$$F = f(\theta_1)\ |1 + xe^{-j\alpha}|$$

$$= f(\theta_1)\ |\sqrt{1 + x^2 + 2x \cos \alpha}| , \tag{8}$$

in which

$$\alpha = \frac{2\pi\delta}{\lambda} + \phi , \tag{9}$$

where ϕ is the phase angle of the complex reflection coefficient. This phase angle is defined as the phase *lag* of the reflected wave relative to the phase of the incident wave.

Path Difference and Divergence Factor

If the earth were flat, it is readily shown that in the terms of Fig. 1,

$$\delta \approx \frac{2h_1h_2}{d} , \tag{10}$$

where d is the horizontal distance between the antenna and the target. This result is obtained by assuming that $d^2 >> h_i^2$. The slant range, R, is given by

$$R = \sqrt{(h_2 - h_1)^2 + d^2} . \tag{11}$$

When the heights h_1 and h_2 and the distance d are such that the earth's curvature is significant, Eqs. (10) and (11) are not really correct, although in many practical situations they are excellent approximations. The exact curved-earth solution for δ is complicated, because it requires solution of a cubic equation. The method that has been used for the computer programs being described is that given by Kerr (1), p. 113 ff, in terms of a pair of parameters S and T. These parameters are defined by the following expressions, in which h_1 and h_2 are now (and henceforth) defined as heights above the earth's surface rather than above the tangent plane as in Fig. 1:

$$S = \frac{d}{\sqrt{2a_e}\,(\sqrt{h_1} + \sqrt{h_2})} , \tag{12}$$

$$T = \sqrt{h_1/h_2} \text{ or } \sqrt{h_2/h_1} , \tag{13}$$

in which a_e is the "effective earth's radius," for the assumed conditions of atmospheric refraction, and the choice of definition of T is made to result in $T \leq 1$. The effective earth's radius concept is based on the treatment of atmospheric refraction given by Schelleng, Burrows, and Ferrell (2); it is an approximate method which is adequate for the situation being considered, for antenna and target heights not in excess of a few thousand feet. The accepted value of a_e for "standard refraction" is 4/3 times the actual earth's radius. If the actual radius is taken to be 3440 nautical miles, and if d is in nautical miles and h_1 and h_2 are in feet, Eq. (12) becomes:

$$S = \frac{d}{1.2287\,(\sqrt{h_1} + \sqrt{h_2})} . \tag{14}$$

In terms of these parameters, a correction factor J can be computed, as can also the divergence factor D of Eq. (5). This correction factor is applied to the flat-earth formula for δ so that it becomes

$$\delta \approx \frac{2h_1h_2}{d}\, J(S,T) . \tag{15}$$

Kerr presents curves for J and D as functions of S and T. To compute these quantities, it is necessary to employ the intermediate parameters S_1 and S_2, which are related to S and T by

$$S = (S_1 + S_2T)/(1+T) , \tag{16}$$

$$S_2 = \left[\sqrt{(1-S_1^2)^2 + 4S_1^2T^2} + S_1^2 - 1\right]/(2S_1T) . \tag{17}$$

An iterative procedure is used in machine computation to find values of S_1 and S_2 that simultaneously satisfy Eqs. (16) and (17) for given values of S and T. When S_1 and S_2 have thus been found, J and D are given by

$$J = (1-S_1^2)(1-S_2^2) , \tag{18}$$

$$D = \left[1 + \frac{4S_1^2S_2T}{S(1-S_1^2)(1+T)}\right]^{-1/2} . \tag{19}$$

The iteration is performed using the computer subroutine named INVERT. A listing of the Fortran statements comprising this subroutine is given in Appendix B.

Surface-Roughness Factor

The roughness factor r of Eq. (5) is computed as a function of the sea-wave height H, the radar wavelength λ, and the grazing angle ψ, Fig. 1, using a formula given by Ament (3):*

$$r = \exp\left[-2\left(\frac{2\pi H \sin\psi}{\lambda}\right)^2\right] . \tag{20}$$

Beard, Katz, and Spetner (4) have conducted experiments whose results are in approximate agreement with this formula. It was assumed in deriving Eq. (20) that the sea surface has a Gaussian height distribution and that H is the standard deviation. For sea waves that have approximately sinusoidal shape H can be equated to $H'/(2\sqrt{2})$, where H' is the crest-to-trough wave height, since the standard deviation of a sine wave of zero mean value is given by its rms value, which is $1/\sqrt{2}$ times the amplitude of the wave or $1/(2\sqrt{2})$ times the crest-to-trough height.

The grazing angle ψ is given by the formula

$$\tan\psi = \left(\frac{h_1 + h_2}{d}\right) K(S,T) , \tag{21}$$

where the first term is the flat-earth formula and K is a curved-earth correction factor given in terms of S_1, S_2, and T by

$$K = \frac{(1 - S_1^2) + T^2(1 - S_2^2)}{1 + T^2} . \tag{22}$$

Approximate Method for Distant Targets

The spherical-earth computation of δ and ψ using the parameters S and T, as just described, requires *a priori* knowledge of the target height. For plotting coverage diagrams it is convenient to assume a target elevation angle rather than a height. The pattern-propagation factor can be computed as a function of angle if the assumption is made that the target is at a range much greater than the distance from the antenna to the reflection point. The method of doing this for the flat-earth assumption is well known; the path difference in that case becomes simply

$$\delta = 2h_1 \sin\theta_d . \tag{23}$$

This result can be deduced from Fig. 1 by assuming that the earth is flat and the target is so distant that $\psi = \theta_d$. (The lines labeled R and R_2 are then considered to be parallel.)

It is possible to extend this method to apply to the spherical-earth case. The resulting equation for δ is

$$\delta = \sqrt{h_1^2 + a_e(a_e + h)\,\phi^2}\; 2\sin^2(\theta_d + \phi) , \tag{24}$$

*Ament states that this formula was derived, independently, by Pekeris and by MacFarlane, during World War II.

where

$$\phi = \sqrt{[\tan(\theta_d)/3]^2 + 2h_1/(3a_e)} - \tan(\theta_d)/3 \tag{25}$$

and a_e is the effective earth's radius as in Eq. (12). The grazing angle ψ is given by

$$\psi = \theta_d + \phi . \tag{26}$$

The derivation of these results is given in Appendix A.

The calculation of the divergence factor, D, can also be approximated in terms of θ_d by a formula given by Kerr (1), Eq. (474), p. 137:

$$D = \left[\frac{1}{3}\left(1 + \frac{2\zeta}{\sqrt{\zeta^2 + 3}}\right)\right]^{1/2} , \tag{27}$$

where

$$\zeta = \sqrt{\frac{a_e}{2h_1}} \tan\theta_d . \tag{28}$$

Reflection Coefficient of Sea Water

The most common situation that gives rise to an interference pattern is reflection from the sea; therefore it is of importance to evaluate the intrinsic reflection coefficient, ρ_0, and the reflection-coefficient phase angle ϕ for sea water. The equations as given by Kerr (1), p. 396 ff, for the horizontal-polarization coefficient Γ_h and the vertical-polarization coefficient Γ_v, are

$$\Gamma_h = \rho_{0(h)} e^{-i\phi_h} = \frac{\sin\psi - \sqrt{\epsilon_c - \cos^2\psi}}{\sin\psi + \sqrt{\epsilon_c - \cos^2\psi}} , \tag{29}$$

$$\Gamma_v = \rho_{0(v)} e^{-i\phi_v} = \frac{\epsilon_c \sin\psi - \sqrt{\epsilon_c - \cos^2\psi}}{\epsilon_c \sin\psi + \sqrt{\epsilon_c - \cos^2\psi}} , \tag{30}$$

in which ψ is the grazing angle (Fig. 1) and ϵ_c is the complex dielectric constant of the reflecting material, given by

$$\epsilon_c = \epsilon_1 - j\epsilon_2 = \epsilon_1 - j60\lambda\sigma . \tag{31}$$

The real part of ϵ_c is the ordinary dielectric constant, λ is the wavelength, and σ is the conductivity. The factor 60 applies when σ is in mhos per meter and λ is in meters. The values of ϵ_1 and ϵ_2 are functions of frequency.

Figures 2, 3, and 4 are plots of $\rho_{0(h)}$, $\rho_{0(v)}$, and ϕ_v produced by machine computation and machine plotting, using values of ϵ_1 and σ shown in the following table:

f (MHz)	ϵ_1	σ (mhos/meter)
<1500	80	4.3
3000	69	6.5
10000	52	16

These values for $f \leqq 3000$ MHz are taken from Kerr's Table 5.1, p. 398. However, the 10000-MHz value of ϵ_1 is taken from Von Hippel (5). The value at this frequency given by Kerr's table resulted in plots that were in poor agreement with similar plots given by Kerr (his Figs. 5.4, 5.5, and 5.6). It was therefore evident that these curves were calculated for another value of ϵ_1 at $f = 10000$ MHz. This conclusion is substantiated by Kerr's comment on page 401: "The values of ϵ_1 and σ used for these figures were taken from Table 5.1, except for 3 cm" ($f = 10000$ MHz). Using Von Hippel's value of ϵ_1 at this frequency produced better agreement with Kerr's curves. The variation between the values tabulated was approximated by linear expressions, as follows:

$$\left.\begin{aligned} \epsilon_1 &= 80 \\ \sigma &= 4.3 \end{aligned}\right\}, \quad f \leq 1500 \text{ MHz} , \tag{32a}$$

$$\left.\begin{aligned} \epsilon_1 &= 80 - 0.00733\,(f - 1500) \\ \sigma &= 4.3 + 0.00148\,(f - 1500) \end{aligned}\right\}, \quad 1500 < f \leq 3000 , \tag{32b}$$

$$\left.\begin{aligned} \epsilon_1 &= 69 - 0.00243\,(f - 3000) \\ \sigma &= 6.52 + 0.001314\,(f - 3000) \end{aligned}\right\}, \quad 3000 < f \leq 10000 . \tag{32c}$$

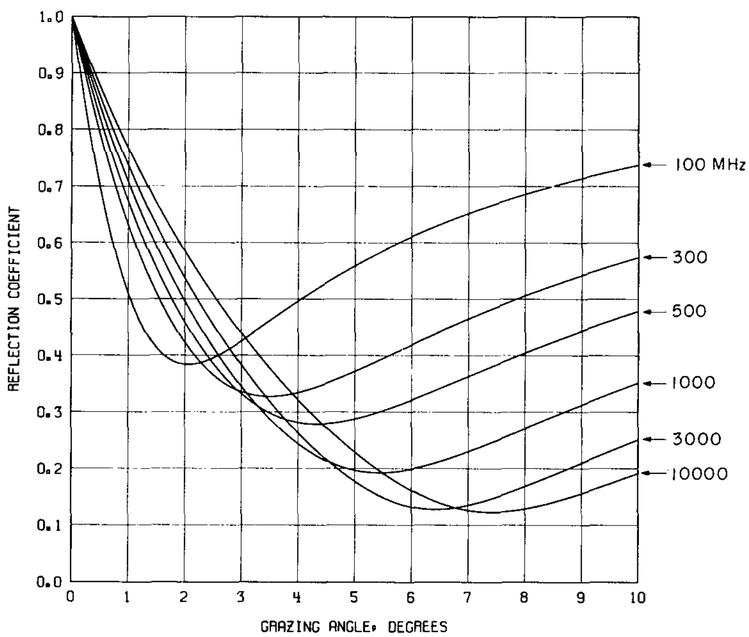


Fig. 2 - Magnitude of the sea-water reflection coefficient for vertical polarization, $\rho_{0(v)}$

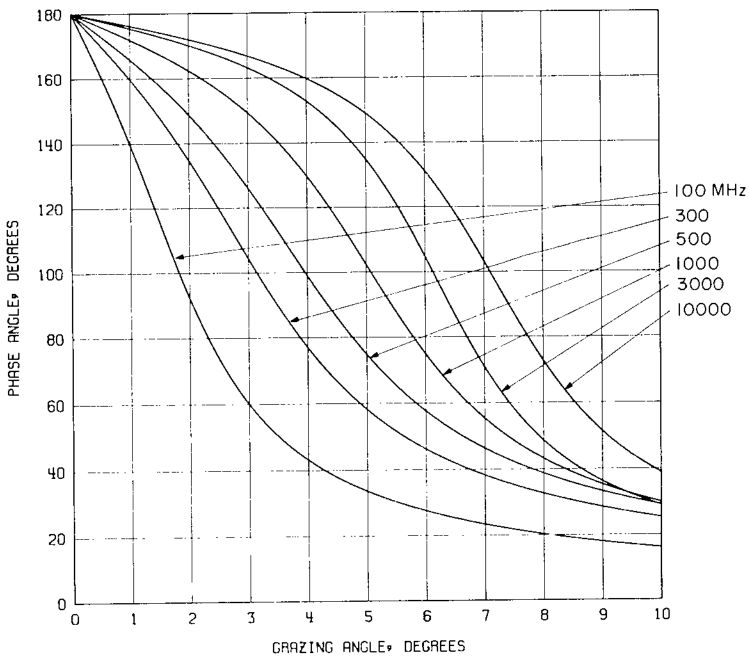


Fig. 3 - Phase angle of the sea-water reflection coefficient for vertical polarization, θ_v

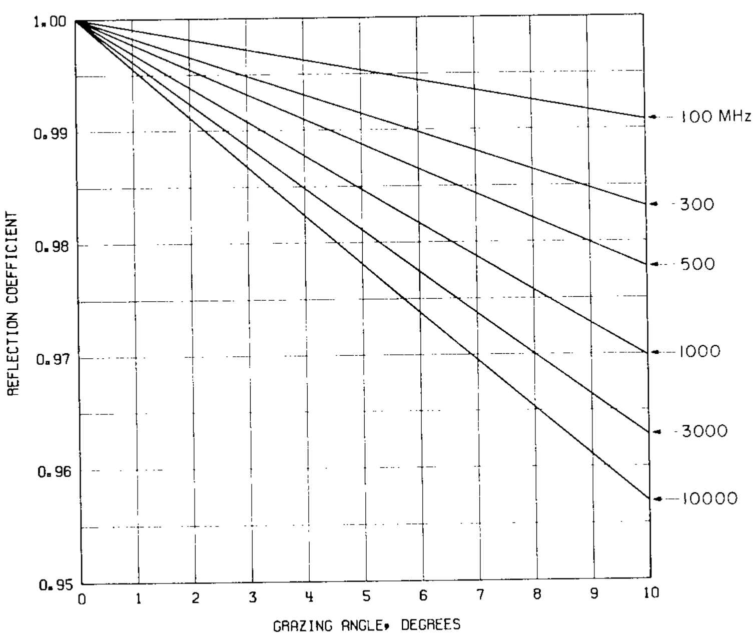


Fig. 4 - Magnitude of the sea-water reflection coefficient for horizontal polarization, $\rho_{0(h)}$

The subroutine used to do the computation of ρ_0 and ϕ for Figs. 2, 3, and 4 is named SEAREF; it is a Fortran coding of Eqs. (29) through (32). A listing of the subroutine is given in Appendix B. Curves are not given for ϕ_h, since the variation from $\phi = 180$ degrees over the range of frequency considered is less than 4 degrees ($\phi_h \approx 184$ degrees at $f = 10000$ MHz and $\psi = 90$ degrees). Also, $\rho_{0(h)} \approx 1$ over this range of frequency and grazing angle. (The variation is magnified in Fig. 4 by the expanded ordinate scale.) The subroutine should not be used for frequencies appreciably above 10000 MHz, because Eqs. (32c) are probably not valid at higher frequencies.

THE INTERMEDIATE REGION

The equations that have been given permit computation of F when there is interference between a direct ray and a reflected ray. This situation exists when a target is above the radar horizon. The target is then said to be in the interference region (Fig. 5). Below the horizon, ray theory predicts a shadow — zero signal strength. There is, however, some signal due to diffraction; consequently the regions above and below the horizon are called, respectively, the interference region and the diffraction region. If the earth had no atmosphere, the only signal in the shadow region would be that due to diffraction. Actually there are also contributions due to scattering by irregularities of the high-altitude atmosphere and, on occasion, there can be strong beyond-the-horizon signals due to anomalous refraction (ducting). The plotting method described here takes into account only the "normal" atmosphere, in which there is no ducting, and it does not consider scattering.*

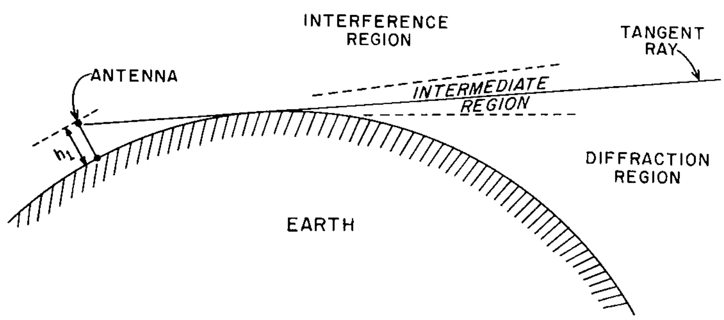


Fig. 5 - Interference, intermediate, and diffraction regions

In the absence of ducting, the field well beyond the horizon at VHF and above is too weak to be of importance for radar. Detection can occur for large targets that are only slightly beyond the horizon with high-sensitivity radars. However, a large increase of power produces only a very small range increase in this region.

In a region very close to the horizon but above it, interference calculations of the type that have been described are not valid. They are based on the principles of ray

*The scatter mechanism can be the chief contributor to the total signal at distances well beyond the horizon (well below the tangent ray). At small distances beyond the horizon, however, the diffraction field predominates. At low frequencies and with vertical polarization, the surface wave can also be of significance, but it is a negligible factor above 100 MHz, especially with horizontal polarization.

optics, which are not valid in this region. A rigorous electromagnetic-wave solution of the problem in this region is very difficult, as discussed by Kerr (1). The mathematical series that describes the field converges very slowly. Further out, well below the horizon, the series converges very rapidly, and the first term adequately describes the diffraction field. This is termed a "one-mode" solution. Thus it is possible to calculate the field strength accurately (if anomalous refraction and scattering are not significant) in both the interference and diffraction regions not too close to the horizon. In the near vicinity of the tangent ray, however, neither the interference calculation nor the one-mode diffraction calculation are valid. Kerr has therefore called this the "intermediate region."

Kerr proposes the following method for estimating the field in the intermediate region; he terms it a method of "bold interpolation." In this method, a plot is made of F expressed in decibels ($20 \log_{10} F$) as a function of the range R for a target whose height is constant, and for a fixed height of the radar antenna. In the interference region this plot will exhibit the maxima and minima, or lobes and nulls, characteristic of the interference of direct and reflected rays. In the diffraction region, $20 \log_{10} F$ decreases monotonically, in a quasi-linear fashion. Kerr's method for plotting F in the intermediate region is simply to draw a smooth curve that joins the curves validly calculated in the interference region, sufficiently above the tangent ray, and in the diffraction region well below the tangent ray, where the one-mode solution is permissible.

The first-mode diffraction-field solution for F can be expressed as the product of three terms (Kerr, p. 122):

$$F = V(X)\, U(Z_1)\, U(Z_2) , \tag{33}$$

in which X, Z_1, and Z_2 are the target range, antenna height, and target height expressed in "natural units." The natural unit of range, L, is given by

$$L = k_1/f^{1/3} , \tag{34}$$

in which f is the radar frequency. For f in megahertz and L in nautical miles, $k_1 =$ 102.7. The natural unit of height, H, is given by

$$H = k_2/f^{2/3} . \tag{35}$$

For f in megahertz and H in feet, $k_2 = 6988$. (See Kerr, pp. 96 and 97, Eqs. (351) and (358).) The natural range X is equal to R/L, where R is the actual range, and the natural height Z is h/H, where h is the actual height.

$V(X)$ is called the attenuation factor and is given by (Kerr, p. 122):

$$V(X) = 2\sqrt{\pi X}\, e^{-2.02X} \tag{36}$$

or in decibels

$$V_{dB} = 10.99 + 10 \log_{10} X - 17.55X . \tag{36a}$$

$U(Z_1)$ and $U(Z_2)$ are called height-gain factors, and their calculation is very complicated (see Kerr, pp. 109-112). However, a curve for $U(Z)$ is given in Kerr, Fig. 2.20, p. 128. In order to be able to use Kerr's plot in a computer program, empirical equations were fitted to the curve. These equations give $U(Z)$ in decibels ($U_{dB} = 20 \log_{10} U$):

$$U_{dB} = 20 \log_{10} Z, \quad Z \leq 0.6, \tag{37a}$$

$$U_{dB} = -4.3 + [51.04 \log_{10} (Z/0.6)]^{1.4}, \quad 0.6 < Z < 1, \tag{37b}$$

$$U_{dB} = 19.85 (Z^{.47} - 0.9), \quad Z \geq 1. \tag{37c}$$

These empirical expressions fit the curves to within about 1 dB. These equations have been incorporated into a Fortran subroutine named UFCN. A listing of the subroutine is given in Appendix B.

These results are valid for horizontal polarization and for the standard 4/3-earth-radius atmosphere. Above 100 MHz, however, the differences in $U(Z)$ for horizontal and vertical polarization are negligible for $Z \geqq 1$. At low to moderate heights, this standard atmosphere is reasonably representative of the actual atmosphere in the absence of ducting.

LOBE PLOTTING

Computer programs for two kinds of plots have been developed, which present the interference lobe patterns in different ways. The first is a plot of radar detection contours in a vertical plane with range and height as coordinates. The second is a plot of signal level in decibels as a function of range for a target of fixed height.

Detection- or Constant-Signal-Strength-Contour Range-Height-Angle Plots

The detection or constant-signal contour plot is made on a coordinate grid called a range-height-angle chart. The basic method of computing this chart has been described (6). For the lobe plots, the chart-plotting program has been made a subroutine, called RHACHT, such that any maximum range and height can be specified. The chart will then be drawn and properly labeled for these specified values. Then a subroutine called LOBES causes the detection contours to be plotted to the proper scale on this chart. The method of plotting the chart is such that refraction is taken into account, on the basis of the CRPL Reference Exponential Atmosphere with $N_s = 313$ (the surface value of the refractive index is 0.000313). (The chart could be plotted for other values of N_s by changing two cards in the RHACHT subroutine.)

The Fortran parameters needed in the calls to these subroutines are:

XMAX — the maximum x-dimension of the chart, inches,

YMAX — the maximum y-dimension,

RMAX — the maximum range of the chart, nautical miles,

HMAX — the maximum height of the chart, feet,

AHFT — the antenna height, feet,

FMHZ — radar frequency, megahertz,

BWD — antenna half-power vertical beamwidth, degrees,

WHFT — sea wave height (crest-to-trough, feet),

TILT — the tilt angle of the antenna beam maximum with respect to the horizon, degrees, THM — the maximum elevation angle to which the plot is to be made,

IPOL — an integer, 1 for vertical polarization, 2 for horizontal,

RFS — the calculated or assumed free-space range of the radar on the specified target, or for a one-way radio system, the free-space range at which the field strength would have a specified value.

The parameters BWD and TILT are needed for calculating the antenna pattern factors, $f(\theta_1)$ and $f(\theta_2)$, Eq. (6). The antenna vertical-plane beam shape is assumed, in this subroutine, to be symmetrical with respect to the beam maximum, and to be of the "$(\sin x)/x$" form, which means that

$$f(\theta) = (\sin u)/u \ , \tag{38}$$

where

$$u = k \sin \theta \text{ radians} \ . \tag{39}$$

The value of k is chosen so that $f(\theta) = 0.7071\ (= 1/\sqrt{2})$ when $\theta = \mathrm{BWD}/2$. This means

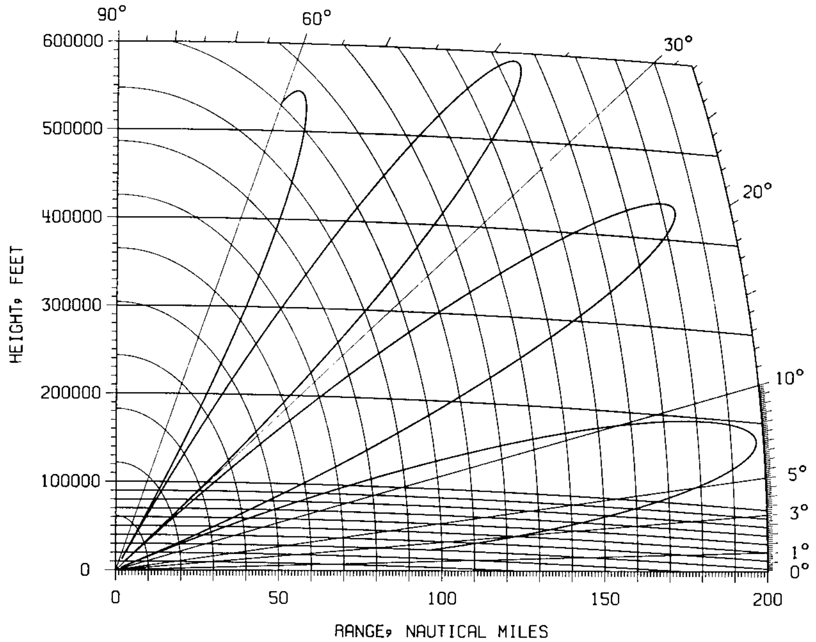
$$k = 1.39157/\sin(\mathrm{BWD}/2) \ . \tag{40}$$

In the plotting subroutine, account is taken of the fact that $f(\theta)$ becomes alternately positive and negative for successive sidelobes of the antenna pattern; when it is negative for the reflected ray and positive for the direct ray, or vice versa, the effect is to add π radians to the phase angle ϕ of the effective reflection coefficient, x, Eq. (7); that is, x becomes negative.

The path difference δ and grazing angle ψ are computed at the lower angles, where earth's curvature is significant, by the method of Eqs. (24) to (27), and D is computed from Eq. (28). The computation is started at the elevation angle for which the path difference is $\delta = \lambda/4$. The path difference is computed by both the flat-earth formula, Eq. (23), and by the curved-earth formula, Eq. (24). (Since these are both long-range approximations, the plotted patterns are not exact in the very-close-range region, but this fact is not usually important.) When an elevation angle is reached for which these two results differ by less than 0.001 wavelength, the flat-earth formula is used thereafter. Also, the value of D is thereafter no longer computed, but is assumed equal to unity, and ψ is assumed equal to the target elevation angle θ_d (Fig. 1).

The basic method of computing the radar free-space range, RFS, is discussed by Kerr (1). The problem is discussed in considerably greater detail in a recent two-part NRL report (7).

Samples of the plots resulting from this program, utilizing subroutines RHACHT and LOBES, are presented in Figs. 6 through 13. These samples show some of the effects that can be studied by varying the frequency, antenna height, beamwidth, beam tilt, sea roughness, and polarization. The first plot, Fig. 6, for a low frequency, low antenna height, broad vertical beamwidth, smooth sea, and horizontal polarization, shows the widely spaced well-defined lobes that are characteristic of this situation, with the lobe maxima reaching to virtually twice the free-space range. The second plot, Fig. 7, shows the effect of simply increasing the antenna height, from 20 feet to 80 feet. This causes the lobes to be finer and more closely spaced, so that there are more of them, and the lowest one lies closer to the surface. The envelope of the lobe maxima is unchanged.



F = 100 MHZ * ANT HT = 20 FT * VERT BW = 90 DEG *
WAVE HT = 0 FT * BM TILT = 0 DEG * FS RNG = 100 NM *
POLARIZATION, HORIZONTAL

Fig. 6 - Detection contour plot for frequency 100 MHz, antenna height 20 ft above sea surface, vertical beamwidth 90 degrees, smooth sea, zero beam tilt, horizontal polarization, and free-space range 100 nautical miles

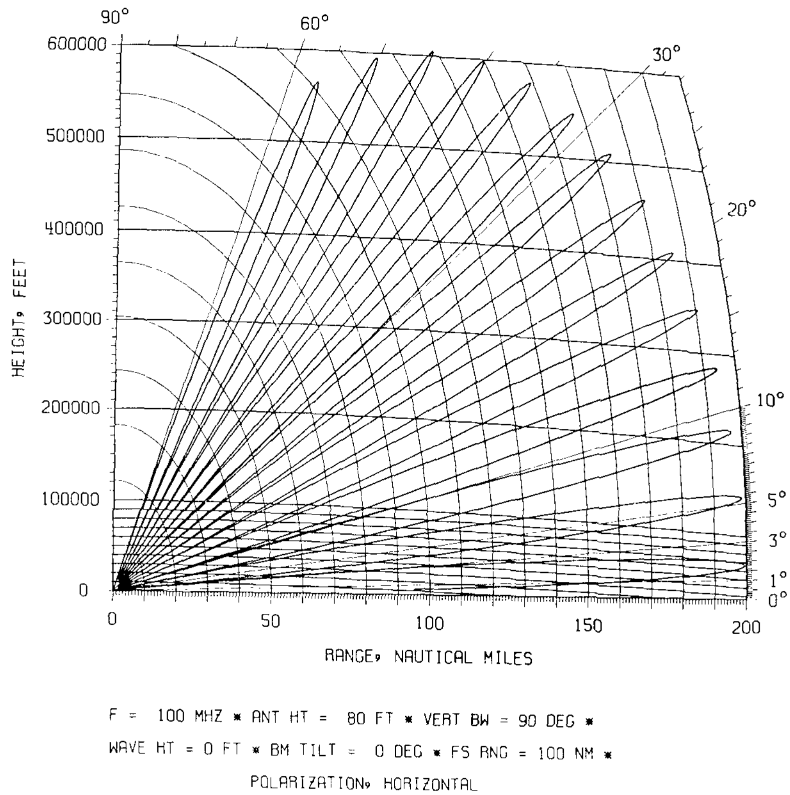


Fig. 7 - Detection contour plot for the same parameters as Fig. 6 except antenna height increased to 80 ft

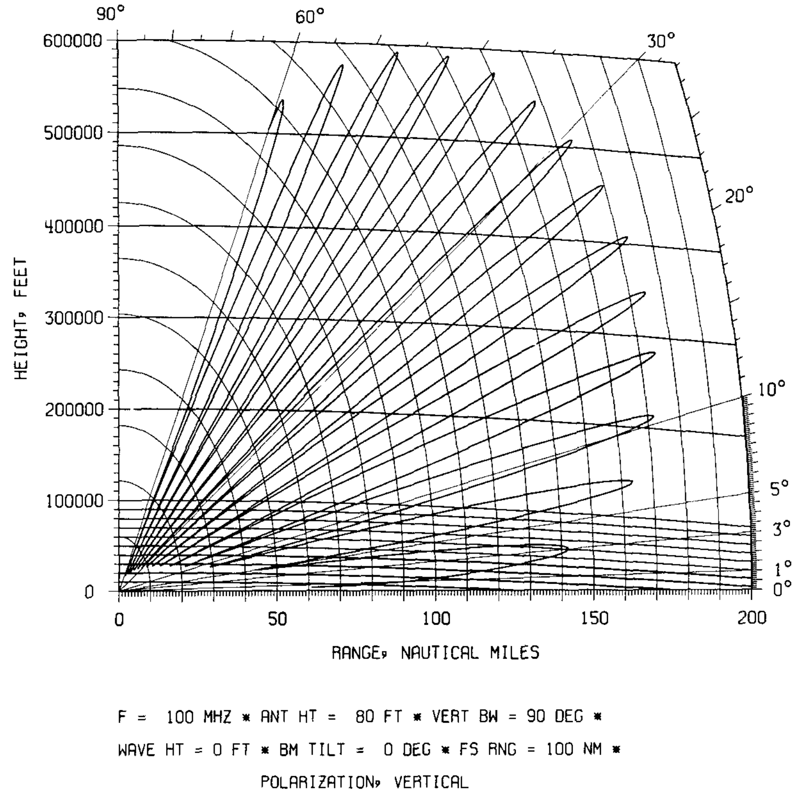


Fig. 8 - Detection contour plot for the same parameters as Fig. 7 except vertical polarization

The third plot, Fig. 8, is for the same conditions as Fig. 7 except that the polarization has been changed from horizontal to vertical. The envelope of the maxima is now radically changed, because of the reduced value of ρ_0 (see Fig. 2). Also, the elevation angles of the lobe maxima are changed, with the lowest lobe at a higher angle; this result is due to the difference in the reflection-coefficient phase angle ϕ, which is 180 degrees for horizontal polarization (see Fig. 3). The superiority of horizontal polarization and the reason for its almost universal use for VHF-UHF radar are clearly seen.

In the next plot, Fig. 9, the polarization is again horizontal, and all other factors except the wave height are the same as in Fig. 7. The effect of 6-foot waves at 100 MHz is seen to be negligible at the lowest elevation angles, but considerable above about 20 degrees. The result is to reduce the magnitude of the reflected wave so that the lobe maxima are of reduced strength and the nulls do not go to near-zero strength, as they do at the lower angles and for a smooth sea. At 60-degree elevation, where the plot is terminated, only a small interference lobe structure remains; the pattern is almost the free-space pattern of the antenna, which is 3 dB down at 45-degree elevation angle (90-degree beamwidth). (The 3-dB pattern factor results in a free-space range of 70.71 miles at this elevation angle.)

In Fig. 10, the frequency has been increased by a factor of 10, to 1000 MHz, the beamwidth has been reduced to 10 degrees, and the free-space range has been decreased to 50 nautical miles. Other factors are the same as in Fig. 7. The principal effect observed is the much finer lobe structure and the much lower angle of the lowest lobe. Also notable is the slightly reduced range of the maximum of the lowest lobe. This is

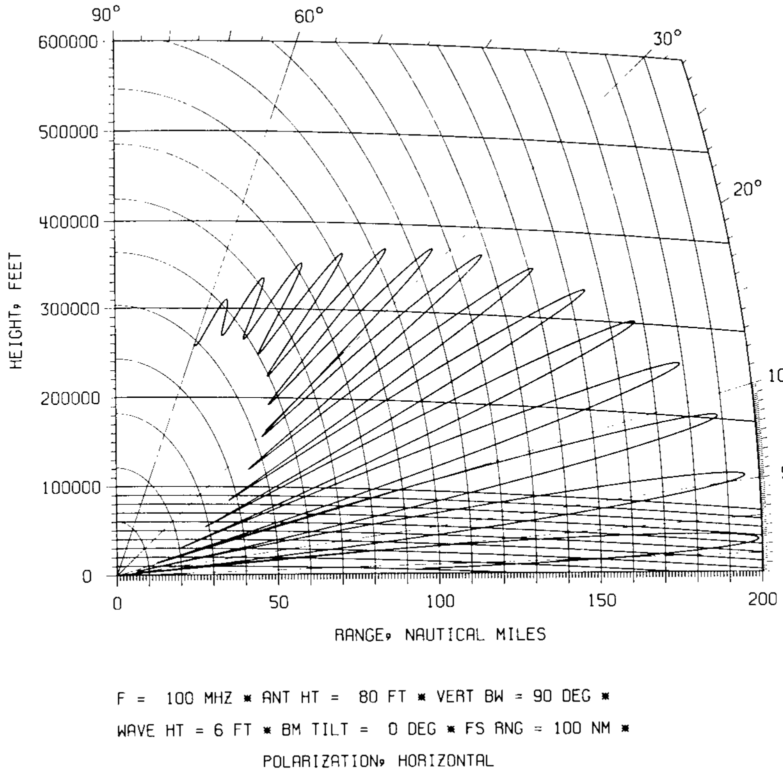


Fig. 9 - Detection contour plot for the same parameters as Fig. 7 except sea wave height increased to 6 ft

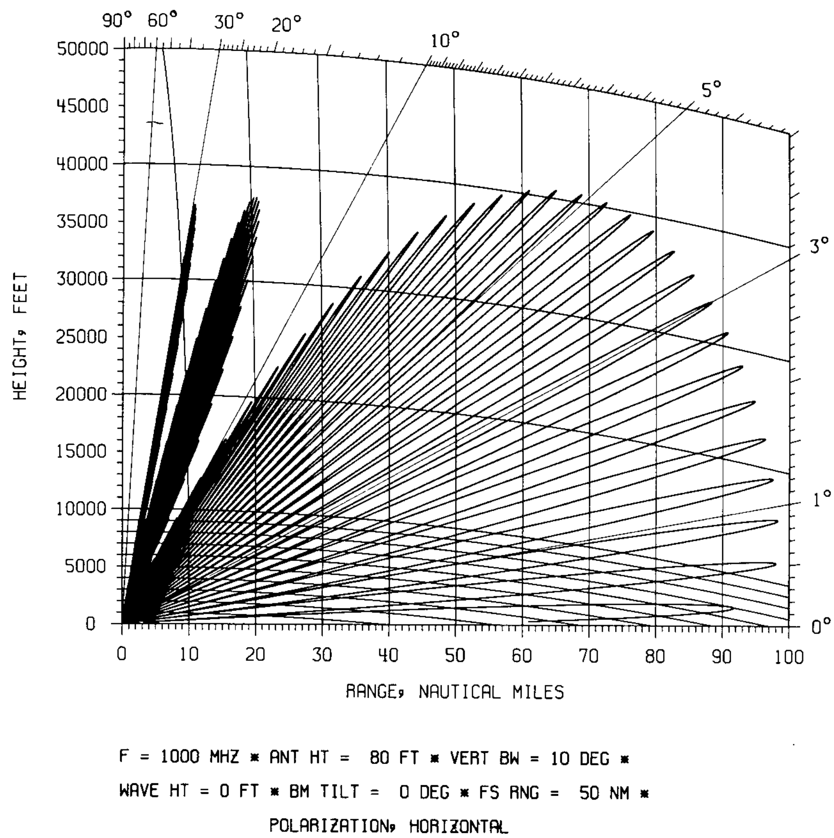


Fig. 10 - Detection contour plot for the same parameters as Fig. 7 except frequency increased to 1000 MHz, antenna vertical beamwidth reduced to 10 degrees, and free-space range decreased to 50 nautical miles

the result of the divergence factor, D, Eqs. (5), (7), and (19), which is negligibly different from unity for the lowest 100-MHz lobe but not so for the lowest lobe at 1000 MHz (with antenna height 80 feet in both cases). Also in this plot the sidelobe pattern of the antenna is visible. The apparent increased fineness of the interference lobes in the antenna-pattern side lobes is not real; it is the result of the nonlinear angle scale of the chart, which is a necessary consequence of the fact that the range and height scales are different.

The next plot, Fig. 11, differs from Fig. 7 only in that the wave height is now 2 feet instead of zero. Again, as at 100 MHz, this has virtually no effect in the very low angle region, but it reduces the lobe strength at higher angles, and fills in the nulls, because of the reduced reflection coefficient. In the first side lobe there is only a faint suggestion of a lobe pattern, and in the second side lobe the pattern is virtually the free-space pattern of the antenna.

Figure 12 shows the effect of changing to vertical polarization at 1000 MHz, again with a smooth sea. As at 100 MHz, the range of detection in the lobe maxima is reduced compared to horizontal polarization, and the lobe angles are changed, but noticeably only in the middle range of angles. The very lowest lobe is only very slightly affected. At the very highest angles (near 90 degrees), the reflection coefficients for vertical and horizontal polarization become nearly equal (exactly equal at 90 degrees since at this angle — vertical incidence — there is no physical distinction between the polarizations).

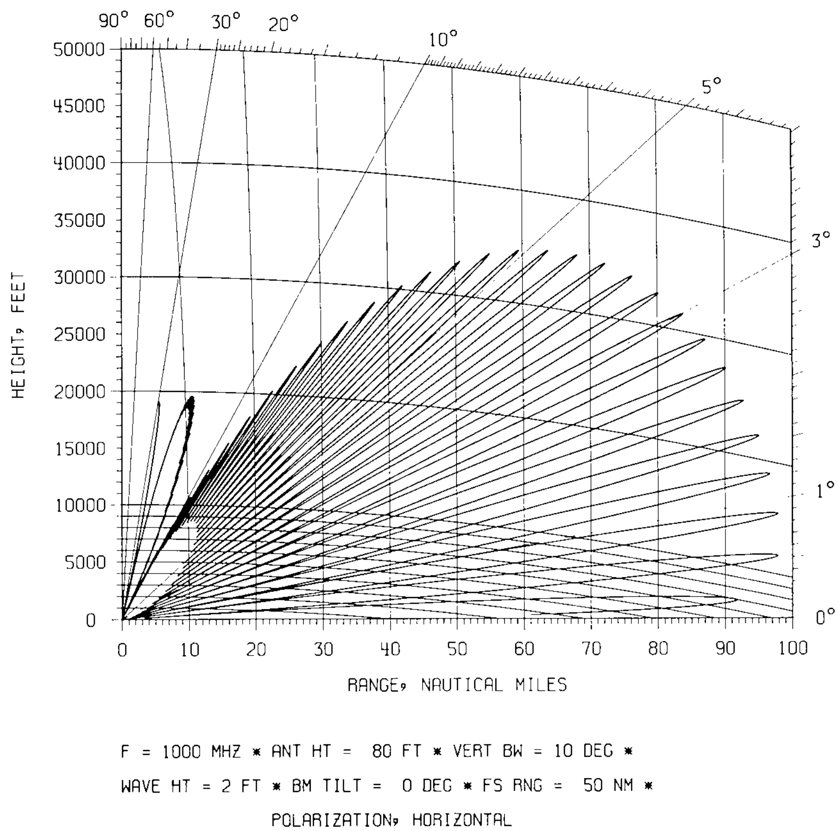


Fig. 11 - Detection contour plot for the same parameters as Fig. 10 except sea wave height increased to 2 ft

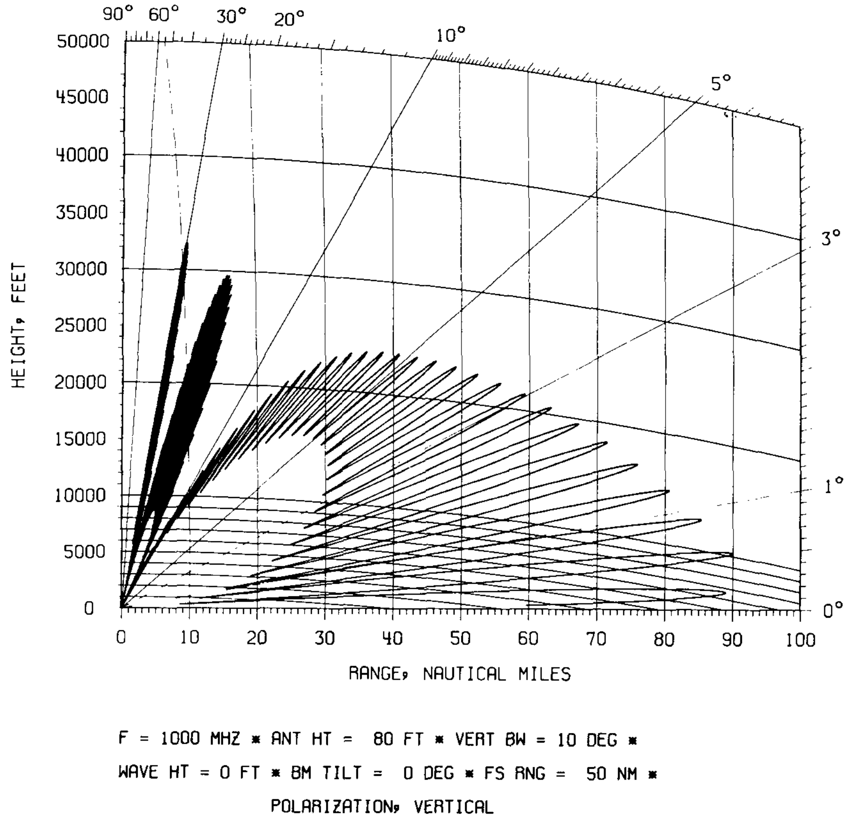


Fig. 12 - Detection contour plot for the same parameters as Fig. 10 except vertical polarization

The last of this series of plots, Fig. 13, is the only one for which the antenna beam is tilted. The tilt is 3 degrees. This causes $f(\theta)$ to be different for the direct and reflected rays, and thus produces strange effects on the interference pattern, especially in the sidelobe region. The disappearance of the lobe structure at about 8-degree elevation is the result of the coincidence of the first null of the free-space antenna pattern with the direction of the reflected ray. (The first pattern null for a 10-degree beamwidth and the assumed $(\sin x)/x$ pattern shape occurs at 11.3 degrees from the beam maximum, which is here elevated by 3 degrees.)

These plots do not show the full contour of the lower side of the lowest lobe. The contour is terminated at an elevation angle slightly greater than zero, because, as discussed in the preceding section, the ray-optical calculation of F is not valid in the "intermediate region." For very-low-altitude targets, when this part of the lobe pattern is important, a second type of plot has been developed.

Signal-Strength-vs-Range Plot for Constant-Altitude Target

In this type of plot, exemplified by Fig. 14, the method described for calculating F in the intermediate region has been used, for the part of the plot below that at which interference-region calculations are valid. Kerr's "bold interpolation" is shown as a dashed extension of the signal-strength curve. The input parameters of the subroutine,

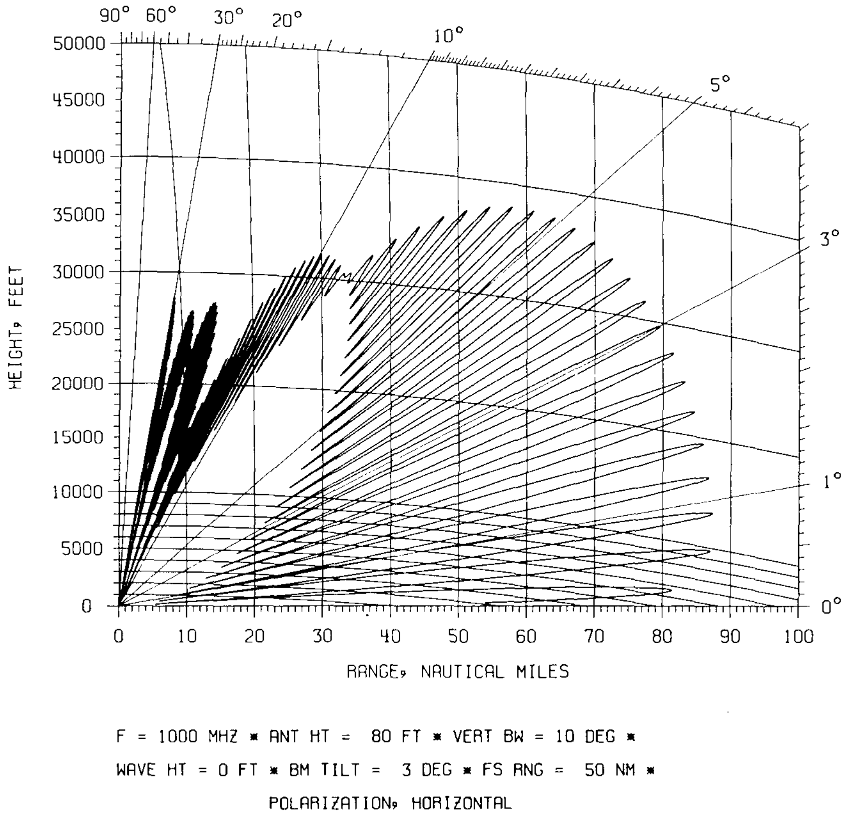


Fig. 13 - Detection contour plot for the same parameters as Fig. 10 except 3-degree beam tilt angle

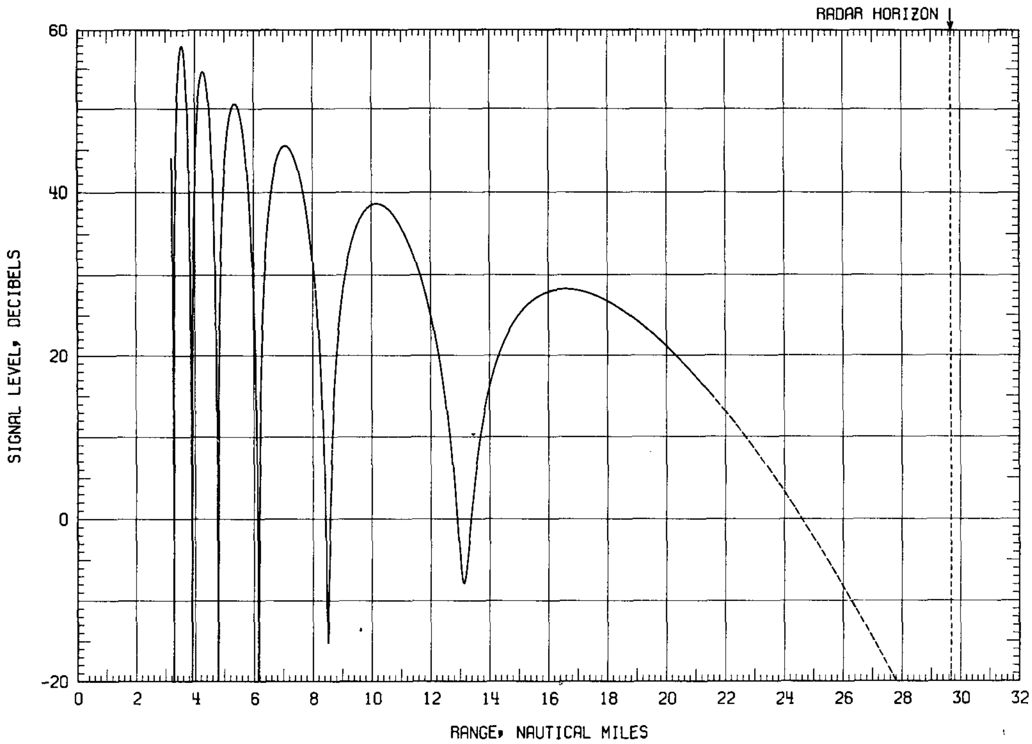


Fig. 14 - Plot of signal strength relative to minimum detectable signal, as a function of range, for frequency 3000 MHz, antenna height 100 ft, target height 200 ft, and free-space detection range 50 nautical miles

named FRPLOT, are (in addition to the x and y dimensions of the plot in inches) the frequency FMHZ (megahertz), the antenna height H1 and the target height H2 (feet), the sea-wave height (crest-to-trough, feet), the polarization (horizontal or vertical), and the calculated or assumed free-space maximum range of the radar RFS (nautical miles) (7). The radar horizon is first calculated from the equation given by Kerr (1),

$$R_h = \sqrt{2a_e}\,(\sqrt{h_1} + \sqrt{h_2}) \tag{41}$$

or

$$R_h = 1.2287\,(\sqrt{h_1} + \sqrt{h_2}) \ , \tag{41a}$$

in which a_e is the "effective earth's radius," here taken to be 4/3 times the actual radius a. For a = 3440 nautical miles, h_1 and h_2 in feet, and R_h in nautical miles, Eq. (41a) applies. (In Eq. (14), incidentally, the denominator is equal to R_h.)

The x-axis coordinate system is then established for a maximum range slightly beyond the horizon. In the interference region, the calculation of F is made from Eqs. (7) through (19). The signal-level computations are made, after calculating F at range R, from the formula

$$S_{dB} = 40\ \log_{10}(FR_0/R) \ , \tag{42}$$

where R_0 is the free-space radar range. This means that the zero-decibel level corresponds to the minimum detectable signal — that is, the signal level in free space at the free-space maximum range. Thus, portions of lobes lying above the zero-decibel line correspond to regions of signal detectability, and those below to regions of no detection. To make a plot of this type for a one-way radio system, it is necessary only to change the factor 40 in Eq. (42) to 20.

Following the recommendation of Kerr (1), the interference-region calculation of F was terminated at the point where the path difference δ of Eqs. (1) and (10) is equal to $\lambda/4$. At greater ranges, the interpolation procedure for the intermediate region was used.

The wave polarization is taken into account in the interference-region calculations, but those for the diffraction region are, strictly speaking, valid only for horizontal polarization. However, for the frequency range being considered the results for horizontal and vertical polarization are not significantly different (Kerr, Ref. 1, p. 124). The antenna pattern is not taken into account because for low-altitude targets (for which this type of plot is primarily intended), the significant radiation all comes from the same part of the beam — the part directed toward the horizon. The free-space range R_0 (Fortran parameter RFS) should be calculated for the free-space antenna gain in that direction, which will usually be the maximum gain.

The signal-level plot is begun at maximum range and is terminated when the range has decreased to one-tenth of the total horizon range, R_h, of Eq. (41).

PLOTTING TECHNIQUES

The plots shown in Figs. 6 through 14 were made by the Gerber Model 875 Automatic Drafting Machine located in the NRL Engineering Services Division. This machine has a maximum plotting area of 5 by 8 feet and a plotting accuracy of 0.005 inch or better, depending on the area of the plot. Standard Leroy India-ink pens are used in a rotatable turret so that more than one pen width can be used in a plot. (The computer plotting subroutines contain pen-changing commands to the plotter.) The sample plots shown were drawn to a size of about 10 by 12 inches, exclusive of lettering, and photographically reduced.

The lettering and numbering is done by the plotter. Subroutines RHACHT and FRPLOT contain the logic required for generating the appropriate numbering for the specified values of RMAX and HMAX or H1 and H2. The plotting and labeling of the signal-level coordinate system (Fig. 14) is done by subroutines named GRID and AXLABL, which are not listed in Appendix B because they are lengthy and not relevant to the primary purpose of this report. The paper tape that controls the plotter is produced by a Fortran subroutine package developed at the Naval Research Laboratory for this purpose; these subroutines are also not listed in Appendix B. The plotter has the capability of drawing a continuous or a dashed line, as illustrated in Fig. 14.

The interpolation required for the dashed-line extension of the signal-level curve in Fig. 14 is done by computing two points in the interference region and two points in the diffraction region, then solving for the cubic equation of a curve that passes through these four points. This equation is then used as the basis for the interpolation — that is, for plotting the dashed section of the curve. This computation is done in a Fortran subroutine named CURVE, of which a listing is given in Appendix B. The subroutine was originally written in a more general form for plotting a smooth curve that passes through any given set of points; that subroutine was named FCURVE, an acronym for "French curve." The CURVE subroutine is a simplification of FCURVE for the special case of just four points. A subroutine named MATALG (matrix algebra) is called by CURVE. Subroutine MATALG solves any set of simultaneous linear equations, as required in this case for obtaining the coefficients of the desired cubic equation. This subroutine was obtained from the NRL Research Computation Center program library; it has the Co-op Identification F2 CODA MATALG.

A special plotting procedure was devised to handle the discontinuities of the curves where they go above or below the maximum or minimum coordinates of the graphs, as in Fig. 14. As each point to be plotted is computed, it is tested to determine whether it lies outside the plotting area; if it does, and if the preceding point does not, then the intersection of the straight line between these points and the boundary of the graph is computed, and the plot is made to that intersection point. The pen is then lifted, and the plotting is resumed at the next intersection of the computed curve with the boundary of the graph. The intersections are computed in a Fortran subroutine called INTRSECT, which is not listed in Appendix B.

Subroutines RHACHT, LOBES, and FRPLOT contain calls to a subroutine named MINITAPE, which is also not listed in Appendix B. Its purpose is to minimize the amount of paper tape used to control the Gerber plotter. It does this by testing each successive computed point to determine whether the curve being plotted has deviated significantly from a straight line since the last point plotted; if it has, the plotter pen is moved to the last preceding computed point, but if not, the pen is not moved (no paper tape punch is made). This technique is possible because the Gerber plotter contains its own interpolation logic to move the pen in a straight line between successive points designated by the paper-tape punches.

The computer time required for these plots, using the CDC 3800, is of the order of a minute or two per plot, depending on the number of lobes in the plot. The plotting time on the Gerber machine, for plots of the kind shown in Figs. 6 through 13, is about 20 minutes per plot. The length of the punched paper tape is generally a few hundred feet. A rough estimate of the computer and plotter charges for each plot is $25.

ACKNOWLEDGMENTS

The author was capably assisted by Frank D. Clarke in the testing of the computer subroutines described. The Fortran subroutine package that produces the punched paper tape for the Gerber plotter was originally written by Arthur T. McClinton, Jr., then of the NRL Space Science Division.

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Appendix A

APPROXIMATE EQUATIONS FOR SPHERICAL-EARTH INTERFERENCE CALCULATION

The approximate formulas for the path difference δ and the grazing angle ψ for spherical-earth reflection are derived as follows. The situation considered is shown in Fig. A1. As indicated, the target is assumed to be so distant that the direct ray and the reflected ray are virtually parallel. The path difference δ is then obviously given by

$$\delta = d_2 - d_1 , \tag{A1}$$

where d_2 is the distance from the antenna to the reflection point and d_1 is the distance that the direct ray travels to reach the point D. This point and the reflection point B are equidistant from the target. The dashed line BD is perpendicular to AD. The problem is to evaluate the path difference δ and the angle ψ as functions of the target elevation angle θ_d and the antenna height h.

It is readily deduced that $\psi = \theta_d + \gamma$, where γ is the angle at the earth's center subtended by the antenna and the reflection point. This follows from the fact that the reflected ray is parallel to the direct ray; its elevation angle with respect to the horizontal at the antenna is therefore θ_d. The horizontal at the reflection point is tilted an amount equal to γ with respect to the horizontal at the antenna.

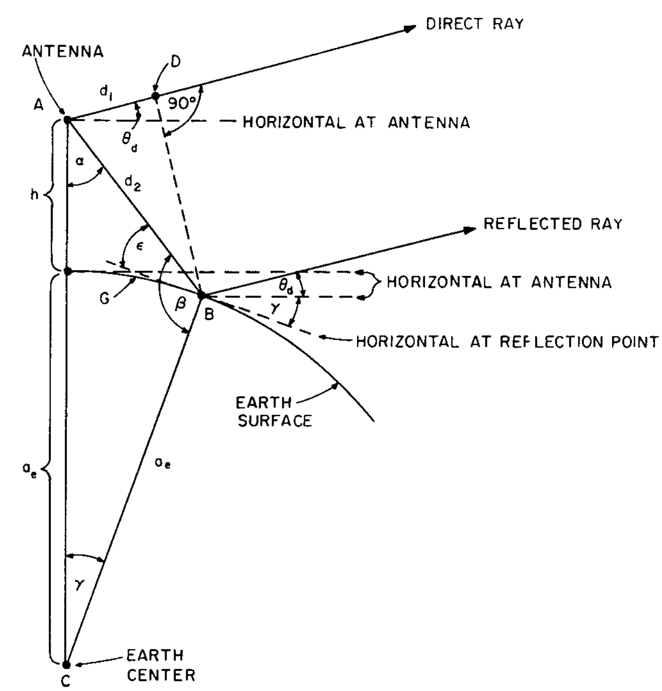

Fig. A1 - Reflection diagram for spherical earth and a distant target

The first problem is to evaluate γ, which can be done by considering the triangle ABC. The angle β is given by

$$\beta = \frac{\pi}{2} + \gamma + \theta_d , \tag{A2}$$

since CB, the earth's radius, is perpendicular to the horizontal at point B, and $\epsilon = \gamma + \theta_d$. Consequently

$$\alpha = \frac{\pi}{2} - 2\gamma - \theta_d . \tag{A3}$$

Applying the law of sines gives

$$\frac{a_e}{\sin\left(\frac{\pi}{2} - 2\gamma - \theta_d\right)} = \frac{a_e + h}{\sin\left(\frac{\pi}{2} + \gamma + \theta_d\right)} . \tag{A4}$$

The trigonometric functions can be rewritten as follows, using the standard formula for the sine of the sum of two angles:

$$\begin{aligned} \sin\left(\frac{\pi}{2} - 2\gamma - \theta_d\right) &= \cos\left(\frac{\pi}{2} - 2\gamma\right) \sin\left(-\theta_d\right) + \sin\left(\frac{\pi}{2} - 2\gamma\right) \cos\theta_d \\ &= -\sin\left(2\gamma\right) \sin\left(\theta_d\right) + \cos\left(2\gamma\right) \cos\left(\theta_d\right) , \end{aligned} \tag{A5}$$

$$\begin{aligned} \sin\left(\frac{\pi}{2} + \gamma + \theta_d\right) &= \cos\left(\frac{\pi}{2} + \gamma\right) \sin\theta_d + \sin\left(\frac{\pi}{2} + \gamma\right) \cos\theta_d \\ &= -\sin\left(\gamma\right) \sin\theta_d + \cos\left(\gamma\right) \cos\left(\theta_d\right) . \end{aligned} \tag{A6}$$

In order to be able to solve for γ, it is necessary to make the following approximations which are valid if $\gamma \ll 1$:

$$\left.\begin{aligned} \sin\left(\gamma\right) &\approx \gamma , \\ \sin\left(2\gamma\right) &\approx 2\gamma , \\ \cos\left(\gamma\right) &\approx 1 - \gamma^2/2 , \\ \cos\left(2\gamma\right) &\approx 1 - 2\gamma^2 . \end{aligned}\right\} \tag{A7}$$

Making these substitutions in Eq. (A4) results in a quadratic equation for γ, which has the solution

$$\gamma = \sqrt{\left[\tan\left(\theta_d\right)/3\right]^2 + 2h/3a_e} - \tan\left(\theta_d\right)/3 . \tag{A8}$$

It is evident that when $(\tan\theta_d/3)^2 >> 2h/3a_e$, γ is a very small angle. To evaluate it accurately for this case, it is preferable to use an approximation which avoids the difficulty of evaluating a small difference between two relatively large numbers; the approximation is based on the relation:

$$\sqrt{1+\epsilon} \approx 1 + \frac{\epsilon}{2} - \frac{\epsilon^2}{8}\,, \quad \epsilon << 1\,, \tag{A9}$$

which is obtained by expanding the left-hand side in a Maclaurin series and retaining only the first three terms. Thus

$$\gamma = (\tan\theta_d/3)\ \sqrt{1 + 6h/(a_e\ \tan^2\theta_d)} - 1 \tag{A8a}$$

$$\approx \frac{h}{a_e\ \tan\theta_d} - \frac{3h^2}{2a_e^2\tan^3\theta_d}\ . \tag{A10}$$

This expression becomes increasingly accurate as θ_d increases, while the "exact" expression is subject to increasing numerical error as θ_d increases, if a fixed number of significant figures is carried in computation. For $h \approx 100$ ft, if as many as ten significant figures are carried, the exact expression is better than the approximation up to $\theta_d = 10$ degrees, while the reverse is true above $\theta_d = 20$ degrees.

When γ has been found, d_2 can be calculated by applying the law of cosines to triangle ABC, again using the approximation $\cos\gamma = 1 - \gamma^2/2$ The result is

$$d_2 = \sqrt{h^2 + a_e(a_e + h)\ \gamma^2}\ . \tag{A11}$$

From Fig. A1 it is evident that

$$d_1 = d_2\ \cos(2\theta_d + 2\gamma) \tag{A12}$$

and therefore

$$\delta = d_2[1 - \cos(2\theta_d + 2\gamma)]$$

$$= 2d_2\ \sin^2(\theta_d + \gamma)\ . \tag{A13}$$

Thus finally

$$\delta = 2\sqrt{h^2 + a_e(a_e + h)\ \gamma^2}\ \sin^2(\theta_d + \gamma)\ . \tag{A14}$$

It can be shown that this equation becomes asymptotically equal to the flat-earth expression, Eq. (23), as $\gamma \to 0$, by the following reasoning. The ground range G from the antenna to the reflection point, Fig. A1, is given by

$$G = \gamma a_e\ . \tag{A15}$$

Therefore, assuming $h << a_e$,

$$h^2 + a_e(a_e + h)\ \gamma^2 \approx h^2 + G^2\ . \tag{A16}$$

If the earth is nearly flat, then

$$h^2 + G^2 \approx d_2^2 \tag{A17}$$

and

$$d_2 \approx h/\sin \theta_d \ . \tag{A18}$$

Therefore, as $\gamma \to 0$,

$$\delta \to 2\sqrt{h^2 + G^2}\ \sin^2 \theta_d \approx 2h\ \sin \theta_d \tag{A19}$$

as was to be shown.

Appendix B

LISTINGS OF FORTRAN SUBROUTINES

In this appendix, the principal Fortran subroutines and function subprograms used for the two lobe-plotting procedures are listed. Some auxiliary subroutines are not listed because they are not relevant to the actual lobe plotting.

The subroutines and subprograms listed are:

Subroutine RHACHT	(1)
Subroutine ATICK	(1)
Function F1	(1)
Subroutine SIMCØN	(1)
Subroutine LØBES	(1)
Subroutine SEAREF	(1,2)
Subroutine FRPLØT	(2)
Subroutine INVERT	(2)
Function ESS	(2)
Subroutine DIFFRACT	(2)
Subroutine UFCN	(2)
Subroutine CURVE	(2)
Subroutine MATALG	(2)

The parenthetic numerals indicate whether the subroutine is used in the range-height-angle coverage-diagram plots (1) or in the constant-target-height signal-level plots (2). Subroutine SEAREF is used in both. The purposes and logic of these subprograms are described by comment cards.

The names and brief descriptions of the subroutines *not* listed are:

Subroutines PLØT, NUMBER, SYMBØL (1,2). These are in the CDC 3800 on-line library plotting subroutine package for the Calcomp plotter. Additional subroutines with entry points named PENCHG, DASHØN, and DASHØFF are used in Gerber plotting.

Subroutine DEGREE (1) plots a degree symbol for labeling the angle scale of the range-height-angle chart.

Subroutine CENTER (1,2) makes a computation required for centering numbers and captions in coordinate labeling.

Subroutine INTRSECT (1,2) computes the intersection point of two straight lines when the x-y coordinates of any pair of points is given for each line.

Subroutine MINITAPE (1,2) checks each computed point of a plot, and makes a call to PLØT if and only if the point will cause the line being plotted to deviate significantly from a straight line. This procedure minimizes the amount of paper tape required for the Gerber plotter

Subroutines GRID and AXLABL (2) plot and label the axes of the coordinate grid used in the constant-target-height signal-level plots.

Subroutine ARRØW (2) plots the arrow that designates the radar horizon line in the signal-level plots.

Listings of these subroutines can be furnished to Government agencies and their contractors upon request. Requests to the author may be addressed as follows: Code 5370, Naval Research Laboratory, Washington, D.C., 20390.

```
      SUBROUTINE RHACHT(XMAX,YMAX,RMAX,HMAX,ANTHITE)
      EXTERNAL F1
      DIMENSION SN(181), RNG1(181), TN1(181), JA(6)
      DIMENSION XZ(180),YZ(180)
      DIMENSION IRFAC (5), IHFAC (8)
      COMMON /A/ REF, GRAD, RAD, CONST, U(182), N
      COMMON /MTPE/ X22,Y22,X11,Y11,XAA,YAA,ERROR
      DIMENSION IANG(9),NANG(9)
      COMMON /B/ XX(181), YY(181), CT1(181), SN1(181), DEL
      COMMON /XCYC/ XCOR,YCOR
      DATA (ERROR = .001)
      DATA ((IANG(IN),IN=1, 9)=1,11,   31,   51,101,111,121,151,181)
      DATA((NANG(IG),IG=1, 9)=0,1,  3,  5,10,20,30,60,90)
      DATA (REF=.000313),(GRAD=.00004384822)
      DATA((IHFAC(I),I=1,8)=50,100,500,1000,5000,10000,50000,100000)
      DATA (( IRFAC (I), I = 1, 5) = 5,10,50 ,100,500)
      DATA ((JA(IA), IA = 1, 6) = 11, 31, 51, 101, 121, 151)
C     FOR PLOTTING RANGE-HEIGHT-ANGLE CHART ON GERBER PLOTTER
C     START WITH HEAVIER OF TWO PENS, IN TURRET POSITION 11
      CALL PENCHG(11)
      DEL=XMAX*.01
      IF (YMAX.LT.XMAX) DEL=YMAX*.01
      BA = 6076.1155*RMAX*YMAX/(HMAX*XMAX)
      BA2 = BA*BA
      PREC = .00001
      CONST = .3048/1852.0
      RAD = 20898950.13 + ANTHITE
      AB = 1.0 + REF
      AB2 = AB*AB
      CD = 2.0 * REF + REF*REF
      II = 0
      DO 29 JI = 1, 2
      IF (JI .EQ. 1) 49, 50
   49 M = 1
      MM = 100
      GO TO 51
   50 M = 11
      MM = 91
   51 DO 29 IK = M, MM
      IL = (IK-1)*10**(JI - 1)
      II = II + 1
      ELEV1 = IL
      ELEV = ELEV1/10.0
      RDN = ELEV/57.29577957
      SN(II) = SIN(RDN)
      S = SN(II)**2
      U(II) = AB2*S-CD
      IF (II .LT. 181) 360, 361
  360 TN = TANF(RDN)
      RDN1 = ATANF(BA*TN)
      TN1(II) = TANF(RDN1)
      CT1(II) = COS(RDN1)
      SN1(II) = SIN(RDN1)
      GO TO 29
  361 CT1(II) = 0.0
      SN1(II) = 1.0
   29 CONTINUE
      B=BA*XMAX
      DO 290 IX= 1,180
```

```
      XZ(IX) = XMAX/(SQRT(1.0 + ((TN1(IX))**2)/BA2))
  290 YZ(IX) = B*SQRT(1.0-(XZ(IX)/XMAX)**2)
C     COMPUTE POINTS ON CONSTANT-HEIGHT CURVES
      H1 = 0.0
      IJM = 100
      JMAX = INTF(ALOG10(HMAX) - .477122)
      HINT = 10.**(JMAX-1) - 1.E-9
      IJQ = (HMAX/(10.**(JMAX-1))+ .00000001)
      DO 30 J = 1, JMAX
      IF (J .EQ. JMAX) IJM = IJQ
      IF (J .EQ. 1) 2, 3
    2 IJ  = 10
      GO TO 4
    3 IJ = 20
    4 DO 30 I = IJ,IJM,10
      H2 = I * 10 ** (J - 1)
      DO 304 K = 1, 181
      N=182-K
      IF (H2 .EQ. 10 .) RNG1(N) = 0.0
      IF (H2 .EQ. 10 . .AND. N .EQ. 1) 6, 7
    6 GAM = REF*GRAD/AB
      RI = 1.0/RAD
      QQ = 2.0*(RI - GAM)
      RNG2 = (CONST*AB/QQ)*2.0*SQRT(QQ*H2)
      GO TO 8
    7 CALL SIMCON (H1,H2,PREC   ,15,RINC,NO1,R,F1)
      RNG2 = RNG1(N) + RINC
    8 RNG1(N) = RNG2
      IF (H2 .LT. HINT) GO TO 304
      A = RNG2*XMAX/RMAX
      B = BA*A
      A2 = A*A
      B2 = B*B
      IF (N .EQ. 181) 46, 47
   46 CALL PLOT (0.,B,3)
      XX(181) = 0.
      YY(181) = B
      X=0.
      Y=B
      X22=0.
      Y22=B
      GO TO 304
   47 XLAST = X
      YLAST = Y
      X = A/(SQRT(1.0+((TN1(N  ))**2)/BA2))
      Y = B*SQRT (1.0 - X*X/A2)
   48 IF (H2 .EQ. HMAX) 87, 68
   87 XX(N)=X
      YY(N)=Y
   68 IF(K .EQ. 2) 780, 781
  780 X11=X
      Y11=Y
      XAA=X
      YAA = Y
      GO TO 304
  781 IF (X .GT. XZ(N) + .0001) 782, 783
  782 CALL INTRSECT (XLAST,YLAST,X,Y, XZ(N),YZ(N),XZ(N+1),YZ(N+1),XO,YO)
      CALL MINITAPE(XO,YO)
      CALL PLOT (XO,YO,2)
```

```
      IF (H2.EQ.HMAX) 785,305
  783 IF (K .EQ. 181) 787, 788
  787 CALL MINITAPE(X,Y)
      CALL PLOT(X,Y,2)
      XCOR = X
      YCOR = Y
      GO TO 304
  788 CALL MINITAPE(X,Y)
  304 CONTINUE
  305 H1 = H2
   30 CONTINUE
      GO TO 789
  785 CALL PLOT(XZ(1),0.,3)
      DO 786 NK=1,N
      XX(NK)=XZ(NK)
      YY(NK)=YZ(NK)
  786 CALL PLOT(XX(NK),YY(NK),2)
      CALL PLOT(X0,Y0,2)
      XCOR = X0
      YCOR = Y0
C     DRAW X-Y AXES
  789 X=0.
      Y = YMAX
      CALL PLOT(X,Y,3)
      Y = 0.0
      CALL PLOT(X,Y,2)
      X=XMAX
      CALL PLOT(X,Y,2)
C     DRAW CONSTANT-RANGE ELLIPSES
C     CHANGE TO FINER PEN
      CALL PENCHG(10)
      KAM=RMAX-1.
      INTR=10
      IF(RMAX.LT. 100.) INTR=5
      IF(RMAX.GT. 300.) INTR=25
      DO 31 KA=INTR,KAM,INTR
      RG = KA
      A = RG*XMAX/RMAX
      B = BA*A
      A2 = A*A
      DO 38 KC = 1, 181
      KR=KC
      IF (KC .EQ. 181) 91, 92
   91 CALL MINITAPE(0.,B)
      CALL PLOT(0.,B,2)
      GO TO 31
   92 X = A/(SQRT(1.0+((TN1(KC))**2)/BA2))
      Y = B*SQRT (1.0 - X*X/A2)
      IF (Y .GT. (YY(KC) + .0001)) 73, 72
   73 IF (KR .EQ. N+1) 730, 731
  730 X1=XCOR
      Y1=YCOR
      GO TO 732
  731 X1=XX(KR-1)
      Y1 = YY(KR-1)
  732 X2=XX(KR)
      Y2 = YY(KR)
      XB=X
      YB = Y
```

```
      CALL INTRSECT(X1,Y1,X2,Y2,XA,YA,XB,YB,X0,Y0)
      CALL PLOT(X0,Y0,2)
      GO TO 31
   72 XA = X
      YA = Y
      IF (KC .EQ. 1) 85, 86
   85 CALL PLOT(X,Y,3)
      X22=X
      Y22=Y
      GO TO 38
   86 IF(KC.EQ.2) 860,861
  860 X11=X
      Y11=Y
      XAA=X
      YAA=Y
      GO TO 38
  861 CALL MINITAPE(X,Y)
   38 CONTINUE
   31 CONTINUE
C     DRAW ANGLE TICK MARKS
  700 CALL ATICK (1,120,1,4,5)
      CALL ATICK (121,181,5,1,2)
C     DRAW RAY LINES
      DO 34 KF = 1, 6
      NF = JA(KF)
      X = 0.0
      Y = 0.0
      CALL PLOT(X,Y,3)
      X = XX(NF)
      Y = YY(NF)
      CALL PLOT(X,Y,2)
   34 CONTINUE
C     MAKE RANGE TICK MARKS ON Y = 0 AXIS
      KT = 9
      DO 364 KY = 1, 800
      KT = KT + 1
      IF (KT .EQ. 10) 368, 369
  368 FAC = 2.0
      KT = 0
      GO TO 370
  369 FAC = 1.0
  370 R2 = KY - 1
      X = R2*XMAX/RMAX
      IF (X .GT. (XMAX + .0001)` GO TO 365
      Y = 0.0
      CALL PLOT(X,Y,3)
      Y = - FAC * DEL
      CALL PLOT(X,Y,2)
  364 CONTINUE
C     MAKE HEIGHT TICK MARKS ON X = 0 AXIS
  365 KS = 9
      KJM = HMAX / HINT + 1.001
      DO 37 KJ = 1, KJM
      KS = KS + 1
      IF (KS .EQ. 10) 375, 376
  375 FAC = 2.0
      KS = 0
      GO TO 377
  376 FAC = 1.0
```

```
  377 H = HINT * (KJ -1)
      Y = H*YMAX/HMAX
      X = 0.0
      CALL PLOT(X,Y,3)
      X = - FAC * DEL
      CALL PLOT(X,Y,2)
   37 CONTINUE
C     LETTER AND NUMBER  RANGE, HEIGHT, AND ANGLE SCALES
C     CHANGE TO HEAVIER PEN
      CALL PENCHG(11)
      IF (XMAX-YMAX) 460,460,461
  460 SFAC=XMAX *.125
      GO TO 462
  461 SFAC=YMAX*.125
  462 H=.175*SFAC
C     SELECT SIZES OF RANGE AND HEIGHT NUMBERING INTERVALS
      DO 100 IR = 1, 5
      NR = RMAX / IRFAC (IR)
      IF (NR .LE. 10) 101, 100
  101 IRUNIT = IRFAC (IR)
      GO TO 102
  100 CONTINUE
      IRUNIT=IRFAC(5)
  102 DO 110 IH = 1, 8
      NH = HMAX / IHFAC (IH)
      IF (NH .LE. 10) 103, 110
  103 IHUNIT = IHFAC (IH)
      GO TO 128
  110 CONTINUE
      IHUNIT=IHFAC(8)
  128 X=-.05*SFAC
      Y=-.5*SFAC
      CALL NUMBER(X,Y,H,0,0.0,2HI1)
      N = IRUNIT
  120 IDIGITS = ALOG10(FLOATF(N)) + 1.000001
      CALL CENTER (H, IDIGITS,4,BIAS)
      X = (N/RMAX) * XMAX - BIAS
      IF (X + BIAS .GT. XMAX) GO TO 801
      CALL NUMBER (X,Y,H,N,0.0,2HI4)
      N = N + IRUNIT
      GO TO 120
  801 Y=-1.0*SFAC
      CALL CENTER (H, 21,21,BIAS)
      X = 0.5 * XMAX - BIAS
      CALL SYMBOL(X,Y,H,21HRANGE, NAUTICAL MILES,0.0,21)
      X=-.5*SFAC
      Y=-.0875*SFAC
      CALL NUMBER(X,Y,H,0,0.0,2HI1)
      N = IHUNIT
      X=-1.1*SFAC
  121 Y=(N/HMAX)*YMAX - .0875*SFAC
      IF (Y .GT. YMAX) GO TO 803
      CALL NUMBER (X,Y,H,N, 0.0,2HI6 )
      N = N + IHUNIT
      GO TO 121
  803 X = -1.40*SFAC
      CALL CENTER(H,12,12,BIAS)
      Y = 0.5 * YMAX - BIAS
      CALL SYMBOL (X,Y,H,12HHEIGHT, FEET,90.,12)
```

```
      XPR= 2.*XMAX
      YPR=-2.0*H
      DO 804 IL = 1, 9
      IND = IANG(IL)
      NAN = NANG(IL)
      CX=.2
      IF (IL .GE. 5) CX=.125
      X=XX(IND) + .4*CT1(IND)*SFAC - CX*SFAC
      Y=YY(IND) + .4*SN1(IND)*SFAC - .0875*SFAC
      IF (IL .EQ.  9) X = X - H
      IF (IL.EQ. 9 .OR. (Y-YPR) .GT. (1.5*H)) GO TO 888
      IF ((XPR-X) .LT. (3.0*H) .OR. X .LT. 2.*H) GO TO 804
  888 CALL NUMBER(X,Y,H,NAN,0.0,2HI2)
      CALL DEGREE (X + .35*SFAC, Y + .175 * SFAC,.08*SFAC)
      XPR=X
      YPR=Y
  804 CONTINUE
      END
```

```
      SUBROUTINE ATICK (IA,JA,KA,MB,MC)
C   THIS SUBROUTINE PLOTS ANGLE TICK MARKS ON RANGE-HEIGHT-ANGLE CHART.
      COMMON /B/ XX(181), YY(181), CT1(181), SN1(181), DEL
      MA = MB
      DO 1 K = IA,JA,KA
      MA = MA + 1
      IF (MA .EQ. MC) 2, 3
    2 FAC = 2.0
      MA = 0
      GO TO 4
    3 FAC = 1.0
    4 X = XX(K)
      Y = YY(K)
      CALL PLOT(X,Y,3)
      X = X + DEL*FAC*CT1(K)
      Y = Y + DEL*FAC*SN1(K)
      CALL PLOT(X,Y,2)
    1 CONTINUE
      END
```

```
      FUNCTION F1(X)
C     INTEGRAND FOR SIMCON SUBROUTINE
      COMMON /A/ REF, GRAD, RAD, CONST, U(182), N
      BB=REF*EXP(-GRAD*X)
      CC=X/RAD
      V=2.0*BB+BB*BB
      W=2.0*CC+CC*CC
      FX = SQRT (U(N) + V + W + V*W)
      F1 = CONST * (1.0 +V)*(1.0 + CC)/FX
      END
```

```
      SUBROUTINE SIMCON(X1,XEND,TEST,LIM,AREA,NOI,R,F)                 EXTRA 3
C     D1 UCSD SIMCON                                                    F 63
C     REVISED OCTOBER 1962       LATER REVISION BY JPM  FEB 1967       SIMCON 3
      NOI=0                                                            SIMCON 4
C     THIS STATEMENT, WHICH READ *R1=10.* REMOVED BECAUSE NOT NEEDED.  SIMCON 5
      ODD=0.0                                                          SIMCON 6
      INT=1                                                            SIMCON 7
      V=1.0                                                            SIMCON 8
      EVEN=0.0                                                         SIMCON 9
      AREA1=0.0                                                        SIMCON10
 19   ENDS=F(X1)+F(XEND)                                               SIMCON11
 2    H=(XEND-X1)/V                                                    SIMCON12
      ODD=EVEN+ODD                                                     SIMCON13
      X=X1+H/2.                                                        SIMCON14
      EVEN=0.0                                                         SIMCON15
      DO 3 I=1,INT                                                     SIMCON16
 21      EVEN=EVEN+F(X)                                                SIMCON17
         X=X+H                                                         SIMCON18
 3    CONTINUE                                                         SIMCON19
 31   AREA=(ENDS+4.0*EVEN+2.0*ODD)*H/6.0                               SIMCON20
      NOI=NOI+1                                                        SIMCON21
 34   R=ABSF((AREA1-AREA)/AREA)                                        SIMCON22
      IF(NOI-LIM)341,35,35                                             SIMCON23
 341  IF(R-TEST)35,35,4                                                SIMCON24
 35   RETURN                                                           SIMCON25
 4    AREA1=AREA                                                       SIMCON26
 46   INT=2*INT                                                        SIMCON27
      V=2.0*V                                                          SIMCON28
      GO TO 2                                                          SIMCON29
      END                                                              SIMCON30
```

```
      SUBROUTINE LOBES (XMAX,YMAX,RMAX,HMAX,FMHZ,AHFT,BWD,WHFT,TILT,THM,
     1 IPOL, RFS)
C THIS SUBROUTINE PLOTS SEA-REFLECTION INTERFERENCE PATTERN FOR RADAR AT
C FREQUENCY FMHZ, ANTENNA HEIGHT AHFT ABOVE SEA WATER.  PLOT IS MADE ON
C RANGE-HEIGHT-ANGLE CHART OBTAINED BY CALLING SUBROUTINE RHACHT WITH MAXIMUM
C DIMENSIONS  XMAX INCHES, YMAX INCHES, MAXIMUM RANGE OF CHART RMAX N. MILES,
C MAXIMUM HEIGHT HMAX FEET.  BWD IS ANTENNA VERTICAL BEAMWIDTH, DEGREES.
C WHFT IS ASSUMED CREST-TO-TROUGH WAVE HEIGHT, FEET.  TILT IS ELEVATION ANGLE OF
C ANTENNA BEAM MAXIMUM, DEGREES.  THM IS MAXIMUM ELEVATION ANGLE TO WHICH LOBE
C PLOT IS DESIRED.  IPOL=1 FOR VERTICAL POLARIZATION, =2 FOR HORIZONTAL.
C RFS IS ASSUMED FREE-SPACE MAXIMUM RANGE OF RADAR FOR SPECIFIED TARGET,
C NAUTICAL MILES (MUST NOT BE GREATER THAN HALF OF RMAX).
C SUBROUTINE WRITTEN BY L. V. BLAKE, CODE 5370, NRL,1968, FOR HORIZONTAL
C POLARIZATION.  MODIFIED SEPT. 1969 TO ALLOW VERTICAL POLARIZATION.
      COMMON /MTPE/  X2, Y2, X11, Y11, XA, YA, ERROR
      DATA (PI=3.141592654),(HALFPI=1.570796327),(PI2=6.283185307),
     1   (RDN=.01745329252),(CONV=1.645788333E-4) ,(AE = 2.786526684E7)
C  COMPUTE ELLIPTICITY, E, OF CONSTANT-RANGE CONTOURS, AND STANDARD
C  DEVIATION, H, OF SINUSOIDAL WAVE OF CREST-TO-TROUGH HEIGHT, WHFT.
      E = YMAX*RMAX/(XMAX*HMAX*CONV)
      H = WHFT*.3535534
C  COMPUTE RADIAN VALUES OF ANGLES GIVEN IN DEGREES.
      TILTR = TILT * RDN
      THMAX = THM * RDN
      BWR = BWD * RDN
C  COMPUTE CONSTANT NEEDED IN EXPRESSION FOR (SIN X)/X ANTENNA PATTERN.
      CONST = 1.39157 / SIN(BWR * .5)
C  COMPUTE WAVELENGTH, W.
      W = 983.573/FMHZ
      W4 = 0.25 * W
      WLIM = 0.01 * W
      FAC = PI2/W
C  COMPUTE ANGLE INCREMENT FOR PLOTTING LOBES.
      DEL1 = 4.92/(FMHZ*AHFT)
      AH2 = AHFT ** 2
      HAE = 2. * AHFT/(3.*AE)
      AEH = AE * (AE + AHFT)
      PARAM = SQRT (AE/(2.*AHFT))
      TUPIW = PI2/W
C  INITIALIZE TARGET ELEVATION ANGLE, THET2.
       THET2 = -DEL1
      N = 0
      INDEX = 0
    1 THET2 = THET2 + DEL1
      IF (THET2 .GE. HALFPI) GO TO 40
      T2 = TANF(THET2)
      S2 = SIN(THET2)
      S3 = S2
      PSI = THET2
C  COMPUTE PATH DIFFERENCE, PD, USING FLAT-EARTH FORMULA.
      PD = 2. * AHFT * S2
      IF (INDEX .EQ. 1) GO TO 77
      T23 = T2/ 3.
C  COMPUTE GRAZING ANGLE, PSI, AND PATH DIFFERENCE, PD1, FOR SPHERICAL
C  EARTH.
      GAM = SQRT (T23**2+HAE)-T23
      PSI = THET2 + GAM
      S3 = SIN(PSI)
      ZETA = PARAM * T2
```

```
      D1= 0.57735 * SQRT(1.0+2.0*ZETA/SQRT(ZETA**2+3.))
      PD1 = SQRT(AH2+AFH*GAM**2)*2.*S3**2
      IF (PD1 .LT. W4) GO TO 1
      N = N+1
      IF (ABS(PD-PD1).LT.WLIM)78,79
C   WHEN FLAT-EARTH, PD, ALMOST EQUALS PD1, THEREAFTER DO FLAT-EARTH
C   APPROXIMATION FOR PD, PSI, AND D (INDEX = 1 CAUSES OMISSION OF
C   SPHERICAL-EARTH CALCULATIONS).
   78 IF (D1 .GE. .999) 780, 79
  780 D1 = 1.
      INDEX = 1
   79 PD = PD1
C   SUBROUTINE SEAREF COMPUTES SMOOTH-SEA COMPLEX COEFFICIENT OF
C   REFLECTION, RHO, PHI, FOR GRAZING ANGLE PSI AND FREQUENCY FMHZ.
   77 CALL SEAREF (FMHZ,PSI,IPOL,RHO,PHI)
C   COMPUTE ROUGHNESS FACTOR.  REFER TO NRL REPORT 6930, EQ. 56.
      RUF = EXP(-8.*(PI*H*S3/W)**2)
C   ANG IS ANGLE THAT DIRECT RAY MAKES WITH DIRECTION OF BEAM MAXIMUM.
      ANG = THET2 - TILTR
      IF (ANG .EQ. 0.) 22, 23
C   COMPUTE PATTERN FACTOR, PAT, FOR DIRECT RAY.
   22 PAT = 1.
      GO TO 24
   23 UU = CONST * SIN (ANG)
      INT = UU/PI2
      DIFF = UU - INT * PI2
      PAT = SIN(DIFF) / UU
C   ANGR IS ANGLE MADE BY REFLECTED RAY WITH DIRECTION OF BEAM MAXIMUM.
   24 ANGR = THET2 + TILTR + 2. * GAM
      IF (ANGR .EQ. 0.) 25, 26
   25 PATR = 1.
      GO TO 27
C   COMPUTE PATTERN FACTOR, PATR, FOR REFLECTED RAY.
   26 UUR = CONST * SIN(ANGR)
      INTR = UUR / PI2
      DIFFR = UUR - INTR * PI2
      PATR = SIN(DIFFR) / UUR
   27 IF (ABS(PAT).LT.1.E-44) 28,29
   28 IF (PAT.LT.0.) 280,281
  280 D = -RHO*RUF*PATR*1.E45*D1
      GO TO 30
  281 D=D1*RHO*RUF*PATR*1.E45
      GO TO 30
C   COMPUTE EFFECTIVE REFLECTION COEFFICIENT.  REFER TO NRL REPORT 6930,
C   EQUATION 62.
   29 D = D1* RHO * PATR/PAT * RUF
C   COMPUTE PHASE DIFFERENCE, ALPHA, AND PATTERN-PROPAGATION FACTOR,F.
   30 ALPHA = FAC * PD + PHI
      INT1 = ALPHA/PI2
      DIFF1 = ALPHA- INT1*PI2
      F = ABS(PAT)* SQRT(1.0 + D*D + 2.0*D*COS(DIFF1))
C   COMPUTE X AND Y COORDINATES CORRESPONDING TO RADAR RANGE (RFS*F) AND
C   ELEVATION ANGLE, THET2.  MOVE PEN TO POINT (X,Y).
      A = RFS*F*XMAX/RMAX
      X = A/SQRT(1.+T2*T2)
      Y =  F * SQRT (A*A-X*X)
      IF (N.EQ.1) 2, 3
    2 CALL PLOT (X,Y,3)
      X2=X
```

```
      Y2=Y
      YP=Y
      XP=X
      GO TO 1
    3 IF (N .EQ. 2) 44, 45
   44 XA=X
      YA=Y
      YP=Y
      XP=X
      X11=X
      Y11=Y
      GO TO 1
C   IF LOBE GOES ABOVE TOP OF CHART, TRUNCATE IT AND DRAW A DASHED LINE
C   AT Y = YMAX.
   45 IF (Y .GT. YMAX) 5, 6
    5 IF (YP .LT. YMAX) 50, 51
   50 CALL INTRSECT(0.,YMAX,XMAX,YMAX,XP,YP,X,Y,XO,YO)
      CALL PLOT(XO,YMAX,2)
   51 YP=Y
      XP=X
      GO TO 1
    6 IF (YP .GT. YMAX) 52, 53
   52 CALL INTRSECT(0.,YMAX,XMAX,YMAX,XP,YP,X,Y,XO,YO)
      CALL PENCHG(4)
      CALL PLOT(XO,YMAX,2)
      CALL PLOT(XO,YMAX,3)
      X2=XO
      Y2=YMAX
      YA=Y
      XA=X
      Y11=YA
      X11=XA
      XP=X
      YP=Y
      GO TO 1
   53 CALL MINITAPF(X,Y)
C   TERMINATE PLOTTING WHEN ELEVATION ANGLE EXCEEDS THMAX.
      IF (THET2 .GE. THMAX) GO TO 40
      XP = X
      YP = Y
      GO TO 1
   40 CALL PLOT(X,Y,2)
      END
```

```
      SUBROUTINE SEAREF (FMHZ, PSI, IPOL, RHO, PHI)
C THIS SUBROUTINE COMPUTES COMPLEX REFLECTION COEFFICIENT OF SEA WATER, AS
C FUNCTION OF FREQUENCY FMHZ IN MEGAHERTZ, GRAZING ANGLE PSI IN RADIANS, WAVE
C POLARIZATION IPOL (1 FOR VERTICAL, 2 FOR HORIZONTAL). OUTPUT IS MAGNITUDE RHO
C AND  PHASE ANGLE PHI (RADIANS) OF COMPLEX COEFFICIENT.  COMPUTATION IS BASED
C ON EQS. (1) AND (2) OF RAD LAB VOL. 13, PAGE 396. SUBROUTINE WRITTEN
C BY L. V. BLAKE NRL CODE 5370 SEPT 1969.
      TYPE COMPLEX EPSC, GAM, SQTERM, TERM
      DATA (FLAST = 0.)
      SINPSI = SIN (PSI)
      CSPSI = COS(PSI)**2
      IF (FMHZ .EQ. FLAST) GO TO 200
C IF SUBROUTINE HAS BEEN CALLED PREVIOUSLY DURING PROGRAM, AND FREQUENCY IS SAME
C AS ON LAST PREVIOUS CALL, PART OF COMPUTATION NEED NOT BE DONE SINCE REQUIRED
C VALUES HAVE BEEN STORED.
      FLAST = FMHZ
C COMPUTE WAVELENGTH
      W = 299.793 / FMHZ
      IF (FMHZ .LE. 1500.) 150, 151
C SIG IS CONDUCTIVITY, MHO/METER, EPS1 IS REAL PART OF DIELECTRIC CONSTANT, BOTH
C DEPENDENT ON FMHZ.
  150 SIG = 4.3
      EPS1 = 80.
      GO TO 155
  151 SIG = 4.3 + (FMHZ - 1500.) * .00148
      IF (FMHZ .LE. 3000.) 153, 154
  153 EPS1 = 80. - (FMHZ - 1500.) * .00733
      GO TO 155
  154 EPS1 = 69. -(FMHZ - 3000.) * .002429
      SIG = 6.52 + (FMHZ - 3000.) * .001314
  155 EPSC = CMPLX (EPS1,-60.*W*SIG)
  200 SQTERM= CSQRT(EPSC-CSPSI)
      IF (IPOL .EQ. 1 ) 160, 161
  160 TERM =     EPSC * SINPSI
      GAM = (TERM-SQTERM)/(TERM+SQTERM)
      GO TO 180
  161 GAM = (SINPSI - SQTERM) / (SINPSI + SQTERM)
  180 RHO = CABS (GAM)
      PHI =-ATAN2 (AIMAG(GAM), REAL (GAM))
      END
```

```
      SUBROUTINE FRPLOT (XMAX,YMAX,H1,H2,WHFT,FMHZ,IPOL,RFS)
C   THIS SUBROUTINE PLOTS RADAR RECEIVED-SIGNAL LEVEL IN DECIBELS ABOVE
C   MINIMUM DETECTABLE, WITH SCALE FROM -20 TO +60, FOR POINT TARGET AT
C   CONSTANT HEIGHT, H2, ABOVE SEA.  RADAR ANTENNA HEIGHT IS H1, SEA WAVE
C   CREST-TO-TROUGH HEIGHT WHFT, FEET, RADAR FREQUENCY FMHZ, MEGAHERTZ.
C   IPOL IS 1 FOR VERTICAL AND 2 FOR HORIZONTAL POLARIZATION , AND RFS IS
C   RADAR FREE-SPACE RANGE IN NAUTICAL MILES.  COORDINATE GRID DIMENSIONS
C   ARE XMAX, YMAX, INCHES.  EARTH CURVATURE EFFECTS COMPUTED BY METHOD
C   DESCRIBED  IN RAD LAB VOL. 13, PP. 113FF.  (KERR, PROPAGATION OF
C   SHORT RADIO WAVES, MC GRAW-HILL, 1951).  ROUGH SEA EFFECTS BY EQ. 56,
C   NRL REPORT 6930.  SMOOTH-SEA REFLECTION BY EQS.(1) AND (2), P. 396,
C   KERR, VALUES OF SEA COMPLEX DIELECTRIC CONSTANT DISCUSSED NRL REPORT
C   7098.  SUBROUTINE WRITTEN BY L.V. BLAKE, CODE 5370, NAVAL RESEARCH
C   LABORATORY.  VERSION OF MARCH 1970.
      EXTERNAL ESS
      COMMON/B/T,TT
      COMMON /MTPE/ X2,Y2,X1,Y1,XA,YA,ERROR
      COMMON /MN/ M1, N1
      COMMON /XYL/ XLAST, YLAST
      DIMENSION XX(4,1), YY(4,1)
      DATA (ERROR = .001)
      DATA (PI = 3.1415926536),(RDN = 57.2957795),(TUPI = 6.281853072),
     1 (PI2 = 1.5707963268)
      YF(Q)=YMAX*(.25+.5*ALOG10(Q))
      INDEX = 0
      FRAC = .99
      TIC = YMAX * .01
      AJ = 0.0
C   CONVERT CREST-TO-TROUGH WAVEHEIGHT TO STANDARD DEVIATION OF SINUSOID.
      WH = WHFT * .35355339
C   COMPUTE W = WAVELENGTH, FEET, AND PARAMETER T.
      W = 983.573/FMHZ
      T = SQRT (H1/H2)
      IF (T .GT. 1.0) T = 1.0/T
      TT = T * T
C   COMPUTE RH = TOTAL HORIZON DISTANCE, NAUT. MI.
      RH = 1.2289 * (SQRT(H1) + SQRT (H2))
      W4 = W * .25
      FAC = 2.0 * H1 * H2/6076.1155
      HH=((H2-H1)/6076.1155)**2
C   DRAW COORDINATE GRID, LABEL AXES.
      NCX = INTF (0.5 * RH+ 1.E-9) + 2
      RMAX = 2. * NCX
      XR=XMAX/RMAX
      XRH = XR * RH
      RTEST = 0.1*RMAX
      CALL GRID (XMAX,YMAX,TIC,TIC,NCX,8,1,1,2,2,1,10,1,10)
      HLETTER = .0175    * YMAX
      XS = XRH - 12. * HLETTER
      YS = YMAX + HLETTER
      CALL SYMBOL (XS, YS, HLETTER, 13HRADAR HORIZON, 0.0, 13)
      YAR = YS + HLETTER
      YAT = YMAX + 0.1 * HLETTER
      TIP = HLETTER * 0.7
      CALL ARROW (XRH, YAR, XRH, YAT, TIP)
      ANTX = 2.
      CALL AXLABL (XMAX,YMAX,HLETTER,0.,ANTX,RMAX,0,1,-20.,20.,60.,0,1,
     1 21HRANGE, NAUTICAL MILES,21,22HSIGNAL LEVEL, DECIBELS,22)
      CALL DASHON
```

```
      SUBROUTINE FRPLOT (XMAX,YMAX,H1,H2,WHFT,FMHZ,IPOL,RFS)
C  THIS SUBROUTINE PLOTS RADAR RECEIVED-SIGNAL LEVEL IN DECIBELS ABOVE
C  MINIMUM DETECTABLE, WITH SCALE FROM -20 TO +60, FOR POINT TARGET AT
C  CONSTANT HEIGHT, H2, ABOVE SEA.  RADAR ANTENNA HEIGHT IS H1, SEA WAVE
C  CREST-TO-TROUGH HEIGHT WHFT, FEET, RADAR FREQUENCY FMHZ, MEGAHERTZ.
C  IPOL IS 1 FOR VERTICAL AND 2 FOR HORIZONTAL POLARIZATION , AND RFS IS
C  RADAR FREE-SPACE RANGE IN NAUTICAL MILES.  COORDINATE GRID DIMENSIONS
C  ARE XMAX, YMAX, INCHES.  EARTH CURVATURE EFFECTS COMPUTED BY METHOD
C  DESCRIBED  IN RAD LAB VOL. 13, PP. 113FF.  (KERR, PROPAGATION OF
C  SHORT RADIO WAVES, MC GRAW-HILL, 1951).  ROUGH SEA EFFECTS BY EQ. 56,
C  NRL REPORT 6930.  SMOOTH-SEA REFLECTION BY EQS.(1) AND (2), P. 396,
C  KERR, VALUES OF SEA COMPLEX DIELECTRIC CONSTANT DISCUSSED NRL REPORT
C  7098.  SUBROUTINE WRITTEN BY L.V. BLAKE, CODE 5370, NAVAL RESEARCH
C  LABORATORY.  VERSION OF MARCH 1970.
      EXTERNAL ESS
      COMMON/B/T,TT
      COMMON /MTPE/ X2,Y2,X1,Y1,XA,YA,ERROR
      COMMON /MN/ M1, N1
      COMMON /XYL/ XLAST, YLAST
      DIMENSION XX(4,1), YY(4,1)
      DATA (ERROR = .001)
      DATA (PI = 3.1415926536),(RDN = 57.2957795),(TUPI = 6.281853072),
     1 (PI2 = 1.5707963268)
      YF(Q)=YMAX*(.25+.5*ALOG10(Q))
      INDEX = 0
      FRAC = .99
      TIC = YMAX * .01
      AJ = 0.0
C  CONVERT CREST-TO-TROUGH WAVEHEIGHT TO STANDARD DEVIATION OF SINUSOID.
      WH = WHFT * .35355339
C  COMPUTE W = WAVELENGTH, FEET, AND PARAMETER T.
      W = 983.573/FMHZ
      T = SQRT (H1/H2)
      IF (T .GT. 1.0) T = 1.0/T
      TT = T * T
C  COMPUTE RH = TOTAL HORIZON DISTANCE, NAUT. MI.
      RH = 1.2289 * (SQRT(H1) + SQRT (H2))
      W4 = W * .25
      FAC = 2.0 * H1 * H2/6076.1155
      HH=((H2-H1)/6076.1155)**2
C  DRAW COORDINATE GRID, LABEL AXES.
      NCX = INTF (0.5 * RH+ 1.E-9) + 2
      RMAX = 2. * NCX
      XR=XMAX/RMAX
      XRH = XR * RH
      RTEST = 0.1*RMAX
      CALL GRID (XMAX,YMAX,TIC,TIC,NCX,8,1,1,2,2,1,10,1,10)
      HLETTER = .0175   * YMAX
      XS = XRH - 12. * HLETTER
      YS = YMAX + HLETTER
      CALL SYMBOL (XS, YS, HLETTER, 13HRADAR HORIZON, 0.0, 13)
      YAR = YS + HLETTER
      YAT = YMAX + 0.1 * HLETTER
      TIP = HLETTER * 0.7
      CALL ARROW (XRH, YAR, XRH, YAT, TIP)
      ANTX = 2.
      CALL AXLABL (XMAX,YMAX,HLETTER,0.,ANTX,RMAX,0,1,-20.,20.,60.,0,1,
     1 21HRANGE, NAUTICAL MILES,21,22HSIGNAL LEVEL, DECIBELS,22)
      CALL DASHON
```

```
C   DRAW FREE-SPACE SIGNAL LEVEL, DASHED.
      DEL = XMAX*.01
      X=XR*RFS*.0316227766
      NN=(XMAX-X)/DEL
      CALL PLOT (X,YMAX,3)
      XLAST = X
      YLAST = Y
      DO 1000 IZ = 1, NN
      X = X + DEL
      R = X/XR
      DB1=40.*ALOG10(RFS/R)
      Y=YMAX*(.25+.0125*DB1)
      IF (Y.LT.0.) 1001,1002
 1001 CALL INTRSECT (XLAST,YLAST,X,Y,0.,0.,XMAX,0.,X0,Y0)
      CALL PLOT (X0,Y0,2)
      GO TO 1003
 1002 CALL PLOT (X,Y,2)
      XLAST = X
      YLAST = Y
 1000 CONTINUE
C   DRAW HORIZON LINE, DASHED.
 1003 CALL PLOT (XRH, YMAX, 3)
      CALL PLOT (XRH, 0., 2)
C   START COMPUTATION AT RANGE 20 PERCENT GREATER THAN THAT AT WHICH PATH
C   DIFFERENCE IS QUARTER WAVELENGTH, OR AT HORIZON, WHICHEVER IS LEAST.
      R  = FAC/W4 * 1.2
      IF (R .GT. RH) R = RH  * .999
      XLAST = XMAX
      YLAST = 1.E-6
    1 IF (AJ - 0.999) 4, 4, 5
    5 IF (D - 0.999) 4, 4, 6
C   WHEN RANGE HAS DECREASED TO POINT AT WHICH EARTH CURVATURE IS
C   NEGLIGIBLE, COMPUTE THEREAFTER ON FLAT-EARTH BASIS.
    6 AJ = 1.0
      D = 1.
      AK = 1.
      GO TO 3
C   COMPUTE PARAMETER S (FRACTION OF TOTAL HORIZON RANGE).
    4 S = R/RH
      S1N = S
      S1M = 1.2 * S
C   FIND PARAMETER S1 BY ITERATION OF FUNCTION ESS, WHICH DEFINES S AS
C   FUNCTION OF S1.
      CALL INVERT (S1N,S1M,4,15,NOI,S1,S,ESS)
      SS1 = S1 * S1
      SQ1 = (1. - SS1) ** 2 + 4. * SS1 * TT
C   COMPUTE S2,D,J,K OF EQS. ON P.115, RAD LAB VOL.13 (D1=D, AJ=J, AK=K).
      S2 = (SQRT (SQ1) - 1. + SS1) /(2. * S1 * T)
      SS2 = S2*S2
      SQ2 = 1.+(4.*SS1*S2*T)/(S*(1.-SS1)*(1.+T))
      D1= 1./SQRT(SQ2)
      AJ = (1.-SS1)*(1.-SS2)
      AK = ((1.-SS1)+TT*(1.-SS2))/(1.+TT)
C   COMPUTE PATH DIFFERENCE, DELTA, AND SLANT RANGE, RSLANT.  AT DELTA =
C   W4, START PLOTTING.
    3 DELTA = FAC/R * AJ
      RSLANT = SQRT (R*R+HH)
      IF (DELTA .LT. W4) 88,89
   89 INDEX=INDEX+1
```

```
C   COMPUTE GRAZING ANGLE, PSI, SMOOTH-SEA REFLECTION COEFFICIENT, RHO,
C   AND PHASE ANGLE, PHI, PHASE DIFFERENCE, ALPHA, ROUGH-SEA COEFFICIENT,
C   RUF, AND PATTERN-PROPAGATION FACTOR, FF.
   90 PSI = ATAN ((H1+H2)/(6076.*R)*AK)
      CALL SEAREF (FMHZ,PSI,IPOL,RHO,PHI)
      RATIO = DELTA / W+PHI/TUPI
      WHOLES = INTF(RATIO)
      RATIO1 = RATIO - WHOLES
      ALPHA = TUPI * RATIO1
      RUF = EXP (-8.*(PI*WH*SIN(PSI)/W)**2)
      D = D1 * RHO * RUF
      FF= SQRT (1.+D*D+2.*D*COS (ALPHA))
      FRR = FF* RFS/RSLANT
      IF (INDEX .GT. 2) GO TO 280
C   SET UP 4-ELEMENT ARRAY OF FIRST TWO POINTS IN INTERFERENCE REGION AND
C   TWO POINTS IN DIFFRACTION REGION (OMIT THIS IF INDEX GREATER THAN 2).
      ISUB=INDEX+2
      XX(ISUB,1)=RSLANT*XR
      YY(ISUB,1)=YF(FRR)
      IF (INDEX .EQ. 1) GO TO 88
      RNM=2.*RH
      CALL DIFFRACT (H1,H2,RNM,FMHZ,FDB)
      FF=10.**(FDB*.1)
      FRR=FF*RFS/RNM
      YY(2,1)=YF(FRR)
      XX(2,1)=RNM*XR
      RNM=1.1*RNM
      CALL DIFFRACT (H1,H2,RNM,FMHZ,FDB)
      FF=10.**(FDB*.1)
      FRR=FF*RFS/RNM
      YY(1,1)=YF(FRR)
      XX(1,1)=RNM*XR
C   DRAW SMOOTH CURVE CONNECTING THE FOUR POINTS, BY CUBIC INTERPOLATION
C   EQUATION.
      CALL CURVE (XX, YY, XMAX, YMAX)
      CALL DASHOFF
C   RESET VALUE OF FRAC FOR PLOTTING INTERFERENCE REGION.
      FRAC = .9998
      GO TO 88
C   COMPUTE X VALUES PROPORTIONAL TO SLANT RANGE, AND Y VALUES
C   PROPORTIONAL TO DECIBELS ABOVE MINIMUM DETECTABLE SIGNAL.
C   STOP PLOTTING IF Y GOES ABOVE OR BELOW GRID BORDER.  RESUME WHEN Y
C   RETURNS TO BORDER.
  280 X = RSLANT * XR
      Y = YF(FRR)
      IF((Y.GT.YMAX.AND.YLAST.GT.YMAX).OR.(Y.LT.0..AND.YLAST.LT.0.))
     1 30,31
   30 XLAST = X
      YLAST = Y
      GO TO 88
   31 IF (Y .LE. YMAX .AND. YLAST .LE. YMAX) GO TO 43
      IF (Y.GT. YMAX .AND. YLAST .LE. YMAX) 40, 41
   40 CALL INTRSECT (0.,YMAX,XMAX,YMAX,XLAST,YLAST,X,Y,XI,YI)
      CALL MINITAPE (XI,YI)
      CALL PLOT (XI,YI,2)
      XLAST = X
      YLAST = Y
      M = 3
      N1 = 0
```

```
      GO TO 88
   41 CALL INTRSECT (0.,YMAX,XMAX,YMAX,XLAST,YLAST,X,Y,XI,YI)
      CALL PLOT (XI,YI,3)
      X2 = XI
      Y2 = YI
      XLAST = X
      YLAST = Y
      N1=2
      M = 2
      GO TO 88
   43 IF (Y .GE. 0. .AND. YLAST .GE. 0.) GO TO 47
      IF (Y .LT. 0. .AND. YLAST .GE. 0.) 44, 45
   44 CALL INTRSECT (0.,0.,XMAX,0.,XLAST,YLAST,X,Y,XI,YI)
      CALL MINITAPE (XI,YI)
      CALL PLOT (XI,YI,2)
      XLAST = X
      YLAST = Y
      M = 3
      GO TO 88
   45 CALL INTRSECT (0.,0.,XMAX,0.,XLAST,YLAST,X,Y,XI,YI)
      CALL PLOT (XI,YI,3)
      X2 = XI
      Y2 = YI
      XLAST = X
      YLAST = Y
      N1=2
      M = 2
      GO TO 88
   47 IF (N1.EQ.2) 473,474
  473 X1=XA=X
      Y1=YA=Y
      N1=3
      GO TO 88
  474 CALL MINITAPE (X,Y)
      XLAST = X
      YLAST = Y
   88 R = FRAC * R
      IF (R.LT.RTEST) 880,1
  880 IF (M.EQ.3) RETURN
      CALL MINITAPE (X,Y)
      CALL PLOT(X,Y,2)
      END
```

```
      GO TO 88
 41   CALL INTRSECT (0.,YMAX,XMAX,YMAX,XLAST,YLAST,X,Y,XI,YI)
      CALL PLOT (XI,YI,3)
      X2 = XI
      Y2 = YI
      XLAST = X
      YLAST = Y
      N1=2
      M = 2
      GO TO 88
 43   IF (Y .GE. 0. .AND. YLAST .GE. 0.) GO TO 47
      IF (Y .LT. 0. .AND. YLAST .GE. 0.) 44, 45
 44   CALL INTRSECT (0.,0.,XMAX,0.,XLAST,YLAST,X,Y,XI,YI)
      CALL MINITAPE (XI,YI)
      CALL PLOT (XI,YI,2)
      XLAST = X
      YLAST = Y
      M = 3
      GO TO 88
 45   CALL INTRSECT (0.,0.,XMAX,0.,XLAST,YLAST,X,Y,XI,YI)
      CALL PLOT (XI,YI,3)
      X2 = XI
      Y2 = YI
      XLAST = X
      YLAST = Y
      N1=2
      M = 2
      GO TO 88
 47   IF (N1.EQ.2) 473,474
473   X1=XA=X
      Y1=YA=Y
      N1=3
      GO TO 88
474   CALL MINITAPE (X,Y)
      XLAST = X
      YLAST = Y
 88   R = FRAC * R      PRINT DIAG & EXIT
      IF (R.LT.RTEST) 880,1
880   IF (M.EQ.3) RETURN
      CALL MINITAPE (X,Y)
      CALL PLOT(X,Y,2)
      END
```

```
      SUBROUTINE INVERT (XMIN,XMAX,NSIG,LIM,NOI,X,FT,F)
C   THIS SUBROUTINE FINDS VALUE OF X THAT RESULTS IN F(X) = FT, BY
C   ITERATION BASED ON LINEAR INTERPOLATION/EXTRAPOLATION FROM PREVIOUS
C   TWO TRIALS.  FUNCTION F MUST BE MONOTONIC.
      TEST = 10.**(-NSIG)
      FD = FT
      IF (FT .EQ. 0.) FD = 1.
      NOI = 1
      X = (XMAX + XMIN)/2.
      F1 = F(X)
      X2 = X
      F2 = F1
      GO TO 4
    1 F1 = F(X)
      IF (NOI .EQ. LIM) RETURN
    4 TEST1 = ABS((F1-FT)/FD)
      IF (TEST1 - TEST) 2, 2, 3
    2 RETURN
    3 XM = X
      IF (NOI .GT. 1) GO TO 6
      DELTA = (XMAX - XMIN)/4.
      IF (FT .LT. F1) DELTA = - DELTA
      FMAX = F(XMAX)
      FMIN = F(XMIN)
      IF (FMAX .LT. FMIN) DELTA = - DELTA
      X = X + DELTA
      XN = XM
      NOI = 2
      GO TO 1
    6 X = (FT-FN)*(X-XN)/(F1-FN) + XN
      NOI = NOI + 1
      IF (NOI - 3) 24, 21, 24
   21 IF ((ABS(F2-FT)) - (ABS(F1-FT))) 23, 23, 24
   23 XN = X2
      FN = F2
      GO TO 1
   24 XN = XM
      FN = F1
      GO TO 1
      END
```

```
      FUNCTION ESS(S1)
C  DEFINES PARAMETER S AS FUNCTION OF S1 AND T. SEE RAD LAB VOL.13,P115.
      COMMON/B/T,TT
      SS1 = S1 * S1
      ESS = (S1+(SQRT((1.-SS1)**2+4.*SS1*TT)-1.+SS1)/(2.0*S1))/(1.+T)
      END
```

```
      SUBROUTINE DIFFRACT (AHFT,THFT,RNM,FMHZ,FDB)
      COMMON/ZZX/Z1,Z2,X,UDB1,UDB2
C  AHFT IS ANTENNA HEIGHT FEET, THFT IS TARGET HEIGHT. RNM IS RANGE NAUTICAL
C  MILES. FMHZ IS FREQUENCY MEGAHERTZ. SUBROUTINE COMPUTES PROPAGATION FACTOR IN
C  DB RELATIVE TO FREE SPACE. BASIS IS EQ. 463 OF ''PROPAGATION OF SHORT RADIO
C  WAVES,''KERR,VOL. 13 OF RAD. LAB. SERIES, PAGE 122. Z1 AND Z2 ARE ''NATURAL
C  HEIGHTS'' AND X IS ''NATURAL RANGE''--EQS.351 AND 358,PAGES 96-97, KERR.
      FACTOR = FMHZ ** .6666667/6988.103
      Z1 = AHFT * FACTOR
      Z2 = THFT * FACTOR
      X = RNM * FMHZ ** .3333333/102.715
      CALL UFCN (Z1,UDB1)
      CALL UFCN (Z2,UDB2)
      FDB=10.99209864 + 10.*ALOG10(X) - 17.545497*X + UDB1 + UDB2
      END
```

```
      SUBROUTINE UFCN (Z, UDB)
C  SUBROUTINE COMPUTES HEIGHT-GAIN FUNCTION UDB IN DECIBELS, BY USING EMPIRICAL
C  FORMULAS FOR DIFFERENT SEGMENTS OF FIG. 2.20, PAGE 128, OF ''PROPAGATION OF
C  SHORT RADIO WAVES,''KERR, VOL. 13 OF RAD. LAB. SERIES.
C IMPORTANT....THIS CURVE IS VALID FOR HORIZONTAL POLARIZATION ONLY.
      IF (Z .LE. .6) 1, 2
    1 UDB=20.*ALOG10(Z)
      RETURN
    2 IF (Z .LT. 1.) 3, 4
    3 UDB = -4.3+51.04*(ALOG10(Z/.6))**1.4
      RETURN
    4 UDB=19.84728*(Z**.47 - .9)
      END
```

```
      SUBROUTINE CURVE (XX, YY, XMAX, YMAX)
      DIMENSION AR(4,4),AY(4,4),XX(4,1),YY(4,1)
      COMMON /XYL/ XPR, YPR
      COMMON /MTPE/ X2, Y2, X1, Y1, XA, YA, ERROR
      COMMON /MN/ M1, N1
      Y(Z) = AY(1,1) + Z*(AY(2,1) + Z*(AY(3,1) + Z*AY(4,1)))
      AR (1,2) = XX(1,1)
      AR(1,3)= XX(1,1)**2
      AR(1,4)=AR(1,3)*XX(1,1)
      AR(2,2)=XX(2,1)
      AR(2,3)=XX(2,1)**2
      AR(2,4)=AR(2,3)*XX(2,1)
      AR(3,2)=XX(3,1)
      AR(3,3)=XX(3,1)**2
      AR(3,4)=AR(3,3)*XX(3,1)
      AR(1,1) = AR(2,1)=AR(3,1)=AR(4,1)=1.0
      AR(4,2)= XX(4,1)
      AR(4,3)=XX(4,1)**2
      AR(4,4)= AR(4,3)*XX(4,1)
      AY(1,1) = YY(1,1)
      AY(2,1)=YY(2,1)
      AY(3,1)=YY(3,1)
      AY(4,1) = YY(4,1)
356   CALL MATALG(AR,AY,4,1,0,DT,4)
      XI = XPR = XMAX
      YI = YPR = Y(XMAX)
      DEL = (XMAX - XX(3,1)) * .1
      IF (YI .LT. 0.) 290, 291
290   DO 292 IJ = 1,10
      XI = XI - DEL
      YI = Y(XI)
      IF (YI .GT. 0.) GO TO 293
      XPR = XI
      YPR = YI
292   CONTINUE
      IF (YI .LT. 0.) RETURN
293   CALL INTRSECT (XPR,YPR,XI,YI,0.,0.,XMAX,0.,XO,YO)
      DEL = (XO - XX(3,1))*0.1
      XI = XO
      YI = Y(XI)
291   CALL PLOT (XI,YI,3)
      XPR=X2=XI
      YPR=Y2=YI
      DO 300 II = 1,10
      XI = XI - DEL
      YI = Y(XI)
      IF (II.EQ.1) 301,302
301   IF (YI.GT.YMAX) 303,304
303   CALL INTRSECT (XPR, YPR, XI, YI, 0., YMAX, XMAX, YMAX, XO, YO)
      CALL PLOT (XO, YO, Z)
      XPR = XI
      YPR = YI
      RETURN
304   X1 = XA = XPR= XI
      Y1 = YA = YPR = YI
      GO TO 300
302   IF (YI .GT. YMAX) 20, 21
 20   CALL INTRSECT (XPR, YPR, XI, YI, 0., YMAX, XMAX, YMAX, XO, YO)
      CALL MINITAPE (XO, YO)
```

```
      CALL PLOT (XO, YO, 2)
      XPR = XI
      YPR = YI
      RETURN
   21 CALL MINITAPE (XI, YI)
  300 CONTINUE
      END
```

```
      SUBROUTINE MATALG(A,X,NR,NV,IDO,DET,NACT)
      DIMENSION A(NACT,NACT),X(NACT,NACT)
      IF(IDO) 1,2,1
    1 DO 3 I=1,NR
      DO 4 J=1,NR
    4 X(I,J)=0.0
    3 X(I,I)=1.0
      NV=NR
    2 DET=1.0
      NR1=NR-1
      DO 5 K=1,NR1
      IR1=K+1
      PIVOT=0.0
      DO 6 I=K,NR
      Z=ABSF(A(I,K))
      IF(Z-PIVOT) 6,6,7
    7 PIVOT=Z
      IPR=I
    6 CONTINUE
      IF(PIVOT) 8,9,8
    9 DET=0.0
      RETURN
    8 IF(IPR-K) 10,11,10
   10 DO 12 J=K,NR
      Z=A(IPR,J)
      A(IPR,J)=A(K,J)
   12 A(K,J)=Z
      DO 13 J=1,NV
      Z=X(IPR,J)
      X(IPR,J)=X(K,J)
   13 X(K,J)=Z
      DET=-DET
   11 DET=DET*A(K,K)
      PIVOT=1.0/A(K,K)
      DO 14 J=IR1,NR
      A(K,J)=A(K,J)*PIVOT
      DO 14 I=IR1,NR
   14 A(I,J)=A(I,J)-A(I,K)*A(K,J)
      DO 5 J=1,NV
      IF(X(K,J)) 15,5,15
   15 X(K,J)=X(K,J)*PIVOT
      DO 16 I=IR1,NR
   16 X(I,J)=X(I,J)-A(I,K)*X(K,J)
    5 CONTINUE
      IF(A(NR,NR)) 17,9,17
   17 DET=DET*A(NR,NR)
      PIVOT=1.0/A(NR,NR)
      DO 18 J=1,NV
      X(NR,J)=X(NR,J)*PIVOT
      DO 18 K=1,NR1
      I=NR-K
      SUM=0.0
      DO 19 L=I,NR1
   19 SUM=SUM+A(I,L+1)*X(L+1,J)
   18 X(I,J)=X(I,J)-SUM
      END
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13. ABSTRACT

Fortran computer subroutines have been developed for machine plotting of radar or radio vertical-plane coverage diagrams for the case of interference between direct and sea-reflected waves. One set of subroutines plots detection or constant-signal-level contours on a range-height-angle chart. A second set plots signal level in decibels as a function of range for a target at a fixed (low) altitude. The first type of plot is valid for antenna heights that are within a few hundred feet of the water and for targets that are at much higher altitudes and not too close to the horizon. The second type is valid for antenna and target heights that are both less than a few thousand feet altitude, with no restriction on minimum altitude. Normal atmospheric refraction is taken into account, but scattering and ducting are assumed to be of negligible effect. The frequency range considered is from about 100 MHz to 10 GHz. The factors taken into account are frequency, antenna and target heights, antenna beamwidth, tilt of the beam with respect to the horizon, roughness of the sea (wave height), polarization of the radio waves, and the calculated or assumed free-space range. Sample plots are shown and discussed. The plotting techniques employed are briefly described, and listings of the essential Fortran subroutines are given in an appendix.

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