1. A discrete-time recurrent network is described by the following equation:

$$\mathbf{y}(n+1) = A \cdot \mathbf{y}(n) + B \cdot \mathbf{x}(n)$$

where

$$A = \begin{bmatrix} 0.7 & 0.5 \\ -0.4 & 0.6 \end{bmatrix} , \quad B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$$

(a) Sketch the dendritic and signal-flow diagrams of the network

(b) Assuming that $\mathbf{y}(0) = 0$ calculate $\mathbf{y}(1)$ and $\mathbf{y}(2)$ for $\mathbf{x}(0) = [1 \;\; 1]^T$ and $\mathbf{x}(1) = [0 \;\; 0]^T$.

(5 + 5 + 5 = 15 marks)

2. A neural network generates its output according to the following equation

$$y = \sigma \left(U \cdot \begin{bmatrix} \boldsymbol{h} \\ 1 \end{bmatrix} \right) , \quad \boldsymbol{h} = \boldsymbol{\sigma} \left(W \cdot \begin{bmatrix} \boldsymbol{x} \\ 1 \end{bmatrix} \right)$$

where σ is a suitable step function (a hard limiter),

$$W = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix} , \quad U = \begin{bmatrix} 2 & 1 & -2 \end{bmatrix} .$$

(a) Sketch a dendritic diagram of the network.

(b) Calculate the network hidden and output signals for the following input vector:

$$\mathbf{x} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

(c) Plot the decision plane/line for every neuron in the network

(d) Write a relationship between inputs and outputs of the modified network in which the step functions σ have been removed.

(5 + 4 + 6 + 5 = 20 marks)

3. Consider an ADALINE with two synapses driven by signals x_1 and $x_2 = 1$.
The training data consists of three input signals: $x_1 = n$ for $n = 1, 2, 3$
and corresponding desired/target outputs: $\mathbf{d} = \begin{bmatrix} 1.2 & 1.8 & 3.2 \end{bmatrix}$

Calculate the gradient of the error function for the weight vector $\mathbf{w} = \begin{bmatrix} 0.3 & -0.2 \end{bmatrix}$

(5 marks)

4. In Adaline, the performance index is given in the following form:

$$J(w_1, w_2) = 4w_1^2 + 6w_1w_2 + 5w_2^2 + 2w_1 - 3w_2 + 2$$

   (a) Determine the cross-correlation and input correlation matrices.
   (b) Assuming that the current weight vector $w = \begin{bmatrix} 1 & -1 \end{bmatrix}$ calculate the gradient of the performance index.
   (c) In the steepest descent learning law, what would be the next value of the weight matrix? Assume that the learning gain $\eta = 1$.
   (d) Assuming that the next values of the input vector $\mathbf{x}(n+1)$ and the desired output $d(n+1)$ are

$$\mathbf{x}(n+1) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} , \quad d(n+1) = 0.5;$$

   calculate the next values of the cross-correlation and input correlation matrices.

($4 + 4 + 1 + 6 = 15$ marks)

5. Consider an ADALINE with three synapses with weights: $\mathbf{w} = [w_1 \;\; w_2 \;\; w_3]$.

   For a given data set, it has been calculated that the eigenvalues of the input correlation matrix are equal to: 0.2, 0.1 and 0.3.

   Determine the maximum stable learning gain using the steepest descent learning law.

(2 marks)

6. A multilayer perceptron calculates its output in the following way:

$$y = \tanh(x_1 + x_2 - 0.5) + 2\tanh(-x_1 + 2x_2 + 1) + 1.5$$

   (a) Determine the weight matrices of the network
   (b) Sketch a dendritic diagram of the network

($3 + 3 = 6$ marks)

7. Consider a feedforward neural network with one hidden layer and a **linear** output layer. Assume that we use a **batch** learning mode.

   In such a case the optimal output weights can be calculated in one step using the normal equation.

   Explain how it can be done giving relevant equations.

(8 marks)

8. Consider a Gaussian RBF neural network with a single output. Derive a steepest descent learning law in a pattern mode.

(12 marks)

9. In a feedforward neural network if we combine the steepest descent algorithm with a line minimization technique than we will be descending the error surface along a zig-zag line, each segment of the line being orthogonal to the previous one.

   Explain in your own words why it is the case. Give relevant equations.

(4 marks)

10. In Generalised Hebbian learning

    (a) the current values of the input vector and the weight matrix are as follows:

    $$\mathbf{x} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} , \quad W = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

    Calculate the weight update ΔW . Assume $\eta = 1$.

    (b) At the conclusion of the learning process, what do the weight matrix, W, and the output vector, y, represent?

$(10 + 4 = 14$ marks)

11. Consider a Kohonen Self-Organizing Map where dimensionality of the input and feature spaces are 2 and 1, respectively. The number of neurons is 8.

    (a) Sketch a structure of the network.

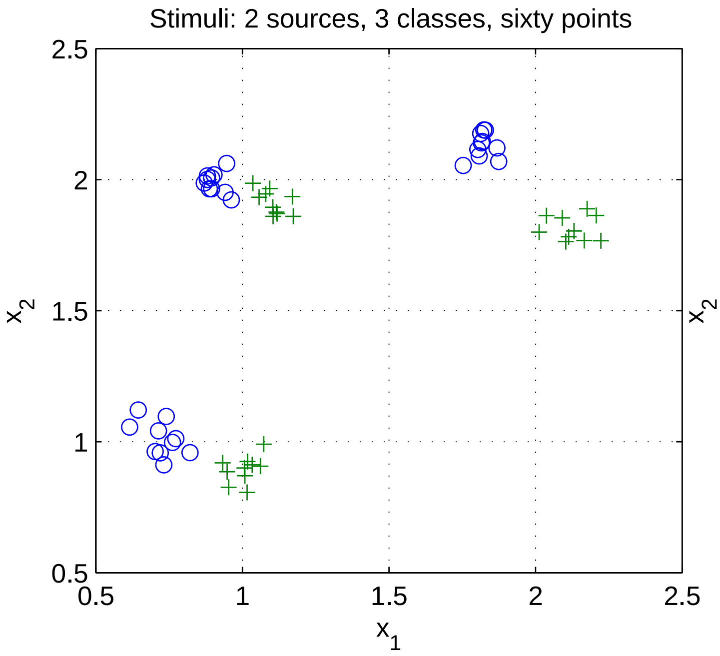
    (b) Assuming the contents of the weight and neuronal position matrices is as follows:

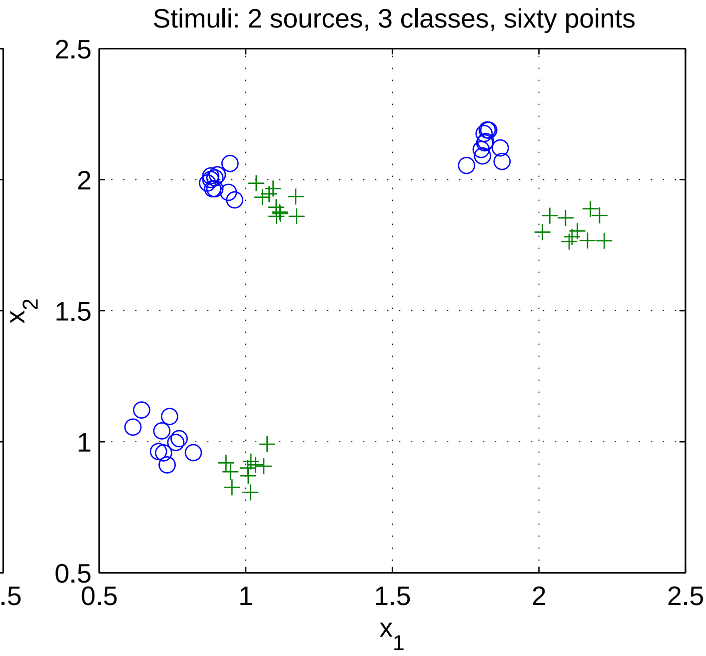
W		V
4	3	5
1	3	1
5	4.5	7
2	1	3
1.5	2	2
4.5	5	6
3	2.5	4
5.5	3	8

    Sketch the resulting feature map.

$(10 + 5 = 15$ marks)

12. The following 2-D data consisting of three pairs of clusters as in the figure below is used to train a Kohonen Self-Organizing Map consisting of 3×3 lattice of neurons. Two copies are provided for your convenience.



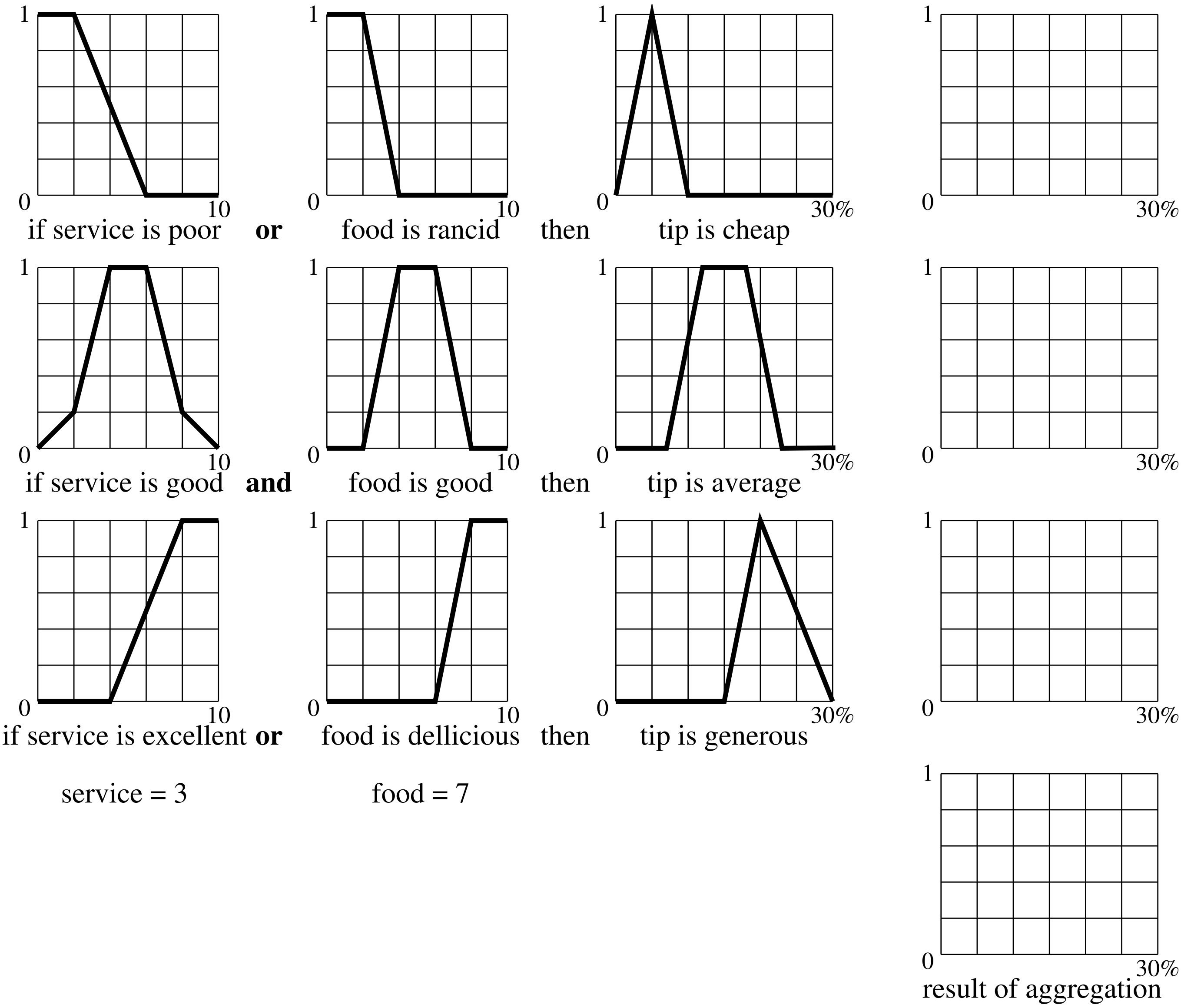


(a) Superimpose on the input data the most likely final feature map.

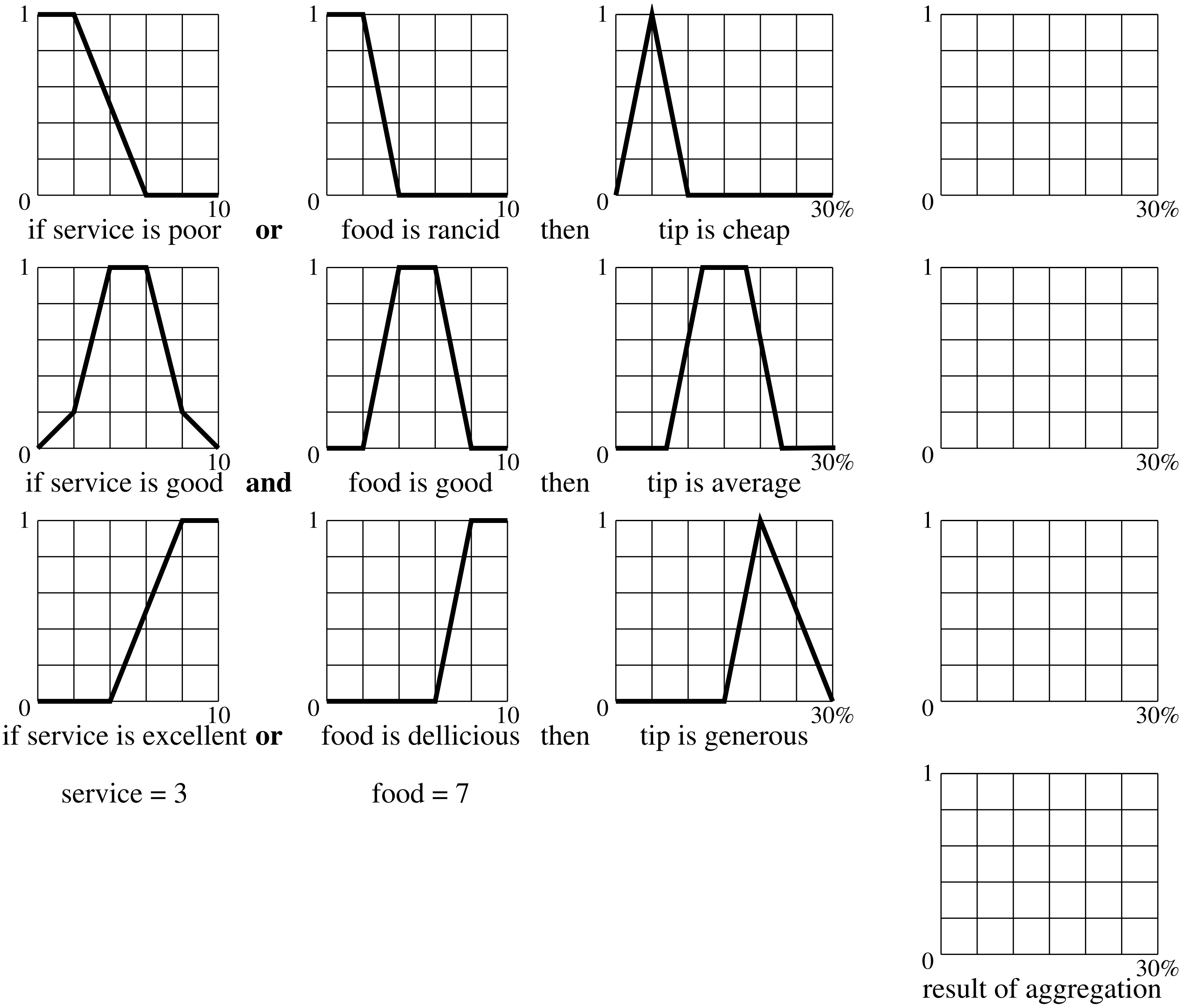
(b) Justify the result.

$(5 + 5 = 10 \text{ marks})$

13. Complete the following Fuzzy inference diagram. Another copy of the diagram is available on the next page. Use it as a draft.



(15 marks)


1
0
10
if service is poor
or
food is rancid
then
tip is cheap
30%
if service is good
and
food is good
then
tip is average
if service is excellent
or
food is dellicious
then
tip is generous
service = 3
food = 7
result of aggregation

Marks do not add up to 100, they add up to 141 instead.