Advanced Concepts For Intelligent Vision Systems (ACIVS'99)

Application of an anisotropic diffusion equation in processing of a class of ophthalmological images.

Andrew P. Papliński
Computer Science and Software Engineering
Monash University, Australia
app@csse.monash.edu.au

James F. Boyce
Wheatstone Laboratory
King's College London, U.K.
jfb@physig.ph.kcl.ac.uk

Abstract

In this paper we discuss application of an anisotropic diffusion equation in processing Posterior Capsular Opacification (PCO) Images. Such images are recorded to monitor the state of a patient's vision after cataract surgery. Non-linear filtering using an anisotropic diffusion equation generates segmentation-like results by enhancing edges represented by the high value of the gradient and smoothing away small inter-regional features. The algorithm ensures an existence of a stable fixed-point solution and maintains a mean grey level of image intensity.

Keywords: Medical Imaging, Posterior Capsular Opacification, Partial Differential Equations, Anisotropic Diffusion Equation, Segmentation.

Address for correspondence:

e-mail: Dr Andrew P. Paplinski
app@csse.monash.edu.au

postal: Dr James F. Boyce
Department of Physics
King's College London
London WC2R 2LS, U.K.
Fax: +44 171 873 2716
ph: +44 171 873 2894

Application of an anisotropic diffusion equation in processing of a class of ophthalmological images.

Abstract

In this paper we discuss application of an anisotropic diffusion equation in processing Posterior Capsular Opacification (PCO) Images. Such images are recorded to monitor the state of a patient's vision after cataract surgery. Non-linear filtering using an anisotropic diffusion equation generates segmentation-like results by enhancing edges represented by the high value of the gradient and smoothing away small inter-regional features. The algorithm ensures an existence of a stable fixed-point solution and maintains a mean grey level of image intensity.

Keywords: Medical Imaging, Posterior Capsular Opacification, Partial Differential Equations, Anisotropic Diffusion Equation, Segmentation.

1 Introduction

Partial differential equations (PDE's) play an increasing role in image processing and analysis due to their algorithmic flexibility. Many theoretical aspects of PDE's and variety of applications have recently been presented in the IEEE Transactions on Image Processing, *Special Issue on Partial Differential Equations ...* ([1]).

In general, if $u(\mathbf{x}, t)$ represents an evolving image, $\mathbf{x}$ being the position vector of a pixel, then the spatial and temporal modification of an image $u(\mathbf{x}, t)$ can be described by the following partial differential equation

$$\frac{\partial u(\mathbf{x}, t)}{\partial t} = \mathcal{D}(\, u(\mathbf{x}, t)\,) \tag{1}$$

where $\mathcal{D}$ is a spatial differentiation operator.

Arguably the most popular example of a partial differential equation used in image processing is given by an anisotropic diffusion equation that we shall discuss in greater detail. Numerous applications of the anisotropic diffusion equation have recently been reported, such as in image filtering and restoration [2, 3]. The majority of considerations related to anisotropic diffusion equations can be traced back to a seminal contribution to the solution of such equations given in [5]. Other PDEs, such as heat equations and wave equations have also been studied in the context of image processing.

In our work we consider a problem related to processing Posterior Capsular Opacification (PCO) images. We shall demonstrate how an anisotropic diffusion equation, similar to that studied in [2], can be used to perform a segmentation-like filtering. Conceptually, this work is linked to our efforts (reported in [6]) of the application of the first order wave equation to model some aspects of an underlying biological process, namely the movement of concentration of epithelial cells which are believed to be responsible for the posterior capsular opacification.

2 Ophthalmological context

The condition of eye lens cataract is ultimately treated by surgery when the patient's natural lens is replaced by an intra-ocular plastic implant [7]. A common post-surgical complication is opacification of the posterior capsule [8]. It can be assumed that Posterior Capsule Opacification is caused by the growth of epithelial cells across the back surface of the capsule, which obscures the implanted lens and again impairs the patient's vision. The opacification is monitored by recording images of the back-surface of the implant at regular intervals after surgery.

An example of a PCO image recorded two years after an operation is given in Figure 1. The diameter of the implanted lens is approximately 6mm, and the image size is in the range of

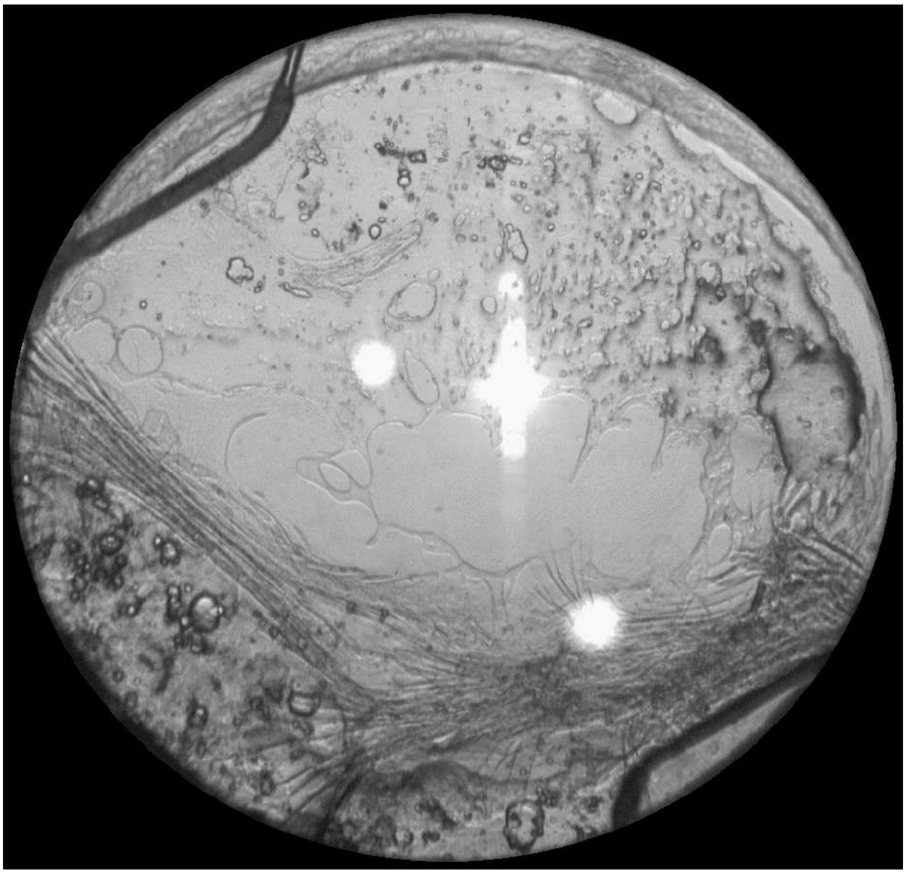

Figure 1: A pre-processed two-year PCO image of a patient with very impaired vision

900×900 pixels. Note the reflection spots characteristic of retro-illumination images.

The Department of Ophthalmology at St. Thomas' Hospital, London and the Image Processing Group from King's College London have developed a software package to assist in the automatic evaluation of posterior capsular opacification and, ultimately, patient's visual acuity. For details the reader is referred to [9, 10, 11, 12]. This paper describes ongoing work on improving and enriching methods of interpretation of PCO images.

3 Image filtering using anisotropic diffusion equations

In this section we shall demonstrate how an anisotropic diffusion equation can be used to filter a PCO image to obtain segmentation-like results. Current methods of segmentation of PCO images are based on co-occurrence arrays as described in [9, 10, 11, 12]. Although the results seem to be satisfactory for practical applications, there is still a non-negligible margin for possible improvements.

Let us consider the following system of partial differential equations (similar to that presented in [2]):

$$\frac{\partial u(\mathbf{x},t)}{\partial t} = \mathrm{div}(L(\mathbf{x},t)\nabla u(\mathbf{x},t)) \tag{2}$$

$$\frac{\partial L(\mathbf{x},t)}{\partial t} + \frac{1}{\tau}L(\mathbf{x},t) = \frac{1}{\tau}F(\nabla u(\mathbf{x},t)) \tag{3}$$

where $u(\mathbf{x},t)$ is an evolving image, $L(\mathbf{x},t)$ is a 2×2 diffusion matrix responsible for anisotropy, and $F(\nabla u(\mathbf{x},t))$ is a 2×2 anisotropy "force" matrix which is a function on an image gradient. The time scale parameter τ is responsible for speed of image evolution.

It can be seen from eqn (3) that the diffusion matrix L is driven by the image gradient, ∇u through the anisotropy force matrix, F.

Often, in similar diffusion models, the image gradient is smoothed by using an appropriately selected filter, as we do in the case of the wave equation. However, with the smoothness assumptions on the anisotropy force matrix, F, namely, that F is a bounded positive-definite matrix with bounded derivatives, the low-pass filtering is in-built into equations (2), (3). For details of the proof the reader is referred to [2].

In order to obtain a segmentation-like filtering of an image we wish to preserve edges which are considered to be significant, and smooth out image features considered to be a noise. One possible choice of the force function, F, which ensures such selective filtering, would be an orthogonal projection on the direction orthogonal to the image gradient. Such a projection will be utilised for the image areas in which the gradient magnitude exceeds a threshold parameter, s. For the image areas with the gradient values below the threshold, the anisotropy force function should ensure smoothing by means of isotropic diffusion. To achieved such processing, the force function is selected to be of the form:

$$F(\nabla u) = \begin{cases} P(\nabla u) & \text{if} \quad |\nabla u| \geq s \\ 1.5(1 - \frac{|\nabla u|^2}{s^2})I + \frac{|\nabla u|^2}{s^2}P(\nabla u) & \text{if} \quad |\nabla u| < s \end{cases} \tag{4}$$

where the projection matrix, $P(\nabla u)$, is:

$$P(\nabla u) = \frac{1}{|\nabla u|^2}\begin{bmatrix} g_2^2 & -g_1 g_2 \\ -g_1 g_2 & g_1^2 \end{bmatrix}, \quad \text{where} \quad \nabla u = [g_1 \; g_2] \tag{5}$$

3.1 Implementation details

Implementation details will be presented in the form of a MATLAB [13] script slightly simplified for brevity.

Let U represent a normalised image, each pixel $u(\mathbf{x}) \in [0\ 1]$. Pixels of the image located outside the lens area are assigned value of 0. We begin with initialisation of the gradient vector and the diffusion tensors:

```
% Initialisation of gradient and anisotropy vector
g = zeros(R,C,2); h = g ;
% Initialisation of diffusion tensor, each Lij = V
V = eye(2);
L = permute(V(:,:,ones(1,R),ones(1,C)),[3 4 1 2]);
% memory allocation for the force tensor F
F = zeros(R,C,2,2) ;
```

Next, we set three important parameters, the sampling time, t_s, the time scale, $\tau = \beta t_s$, responsible for the speed of iterations, and the stiffness threshold, s (image gradient threshold parameter):

```
% sampling time and time scale
ts = 0.2 ; beta = 2; ta = beta*ts ;
           bF=1/(1+beta); bL=beta*bF;
% stiffness threshold (squared)
s2 = .05^2 ;
```

The main iteration loop:

```
for n = 1:N
 % gradient using central differences
 g(:,:,1)=0.5*([U(2:R,:);U(R,:)]-[U(1,:);U(1:R-1,:)]);
 g(:,:,2)=0.5*([U(:,2:C),U(:,C)]-[U(:,1),U(:,1:C-1)]);
 % square of the gradient magnitude
 gm2 = g(:,:,1).^2+g(:,:,2).^2;
```

The pixel loop to determine the diffusion force matrix, F, and to iterate the diffusion tensor:

```
 for r = 1:R
  for c = 1:C
   FM = [g(r,c,2).^2, -g(r,c,1)*g(r,c,2), g(r,c,1).^2];
   if (gm2(r,c) >= s2)
     FM = FM/gm2(r,c) ;
     F(r,c,1,1)= FM(1);
     F(r,c,1,2)= FM(2); F(r,c,2,1)=FM(2) ;
     F(r,c,2,2)= FM(3);
   else
```

```
    FM = FM/s2 ;
    gs2 = 1.5*(1-gm2(r,c)/s2);
    F(r,c,1,1)= FM(1)+gs2;
    F(r,c,1,2)= FM(2); F(r,c,2,1)=FM(2) ;
    F(r,c,2,2)= FM(3)+ gs2;
   end
   % Iteration of the diffusion tensor
   Ln = bL*permute(L(r,c,:,:),[3 4 1 2]) + ...
        bF*permute(F(r,c,:,:),[3 4 1 2]) ;
   L(r,c,:,:) = Ln ;
   % anisotropy vector, h = L g
   h(r,c,:)=Ln*permute(g(r,c,:),[3 1 2]);
  end
 end % of pixel loop
```

Divergence and image iteration:

```
 % divergence of the anisotropy vector using central differences
 dvU = 0.5*([h(2:R,:,1);h(R,:,1)]-[h(1,:,1);h(1:R-1,:,1)] ...
         + [h(:,2:C,2),h(:,C,2)]-[h(:,1,2),h(:,1:C-1,2)]);
 % image update
 U = U + ts*dvU ;
end  % of the main iteration loop
```

The number of iterations required for the image to converge to the steady state is relatively small; $N = 8$, was usually satisfactory. The results are presented on Figures 2 and 3.

The first image in Figure 2 is the original PCO image presented in Figure 1 with two modifications: the reflection spots were patched with noise and the image was subsampled by the factor of 4. It is possible to observe gradual smoothing of high texture regions classified as noise and the relative amplification of well-defined feature borders.

An interesting feature of the diffusion equation in the form (2) is the conservation of the mean grey level of the image intensity.

A "cross-sectional" view along the horizontal line number 60 is shown in Figure 3. The two curves represent the initial and final images. It is possible to note that the anisotropic diffusion maintains prominent edges represented by the high value of the gradient and smoothes inter-regional noise. This is an important property of the algorithm which can be used to enhance existing segmentation algorithms for the PCO images. These images are segmented into smooth areas representing transparent regions of the implanted lens. See [9, 10, 11, 12] for details.

Acknowledgment

The authors would like to express their gratitude to Mr D. J. Spalton from the Department of Ophthalmology of St Thomas' Hospital for the specification of the problem, the provision of the image data and many useful discussions.

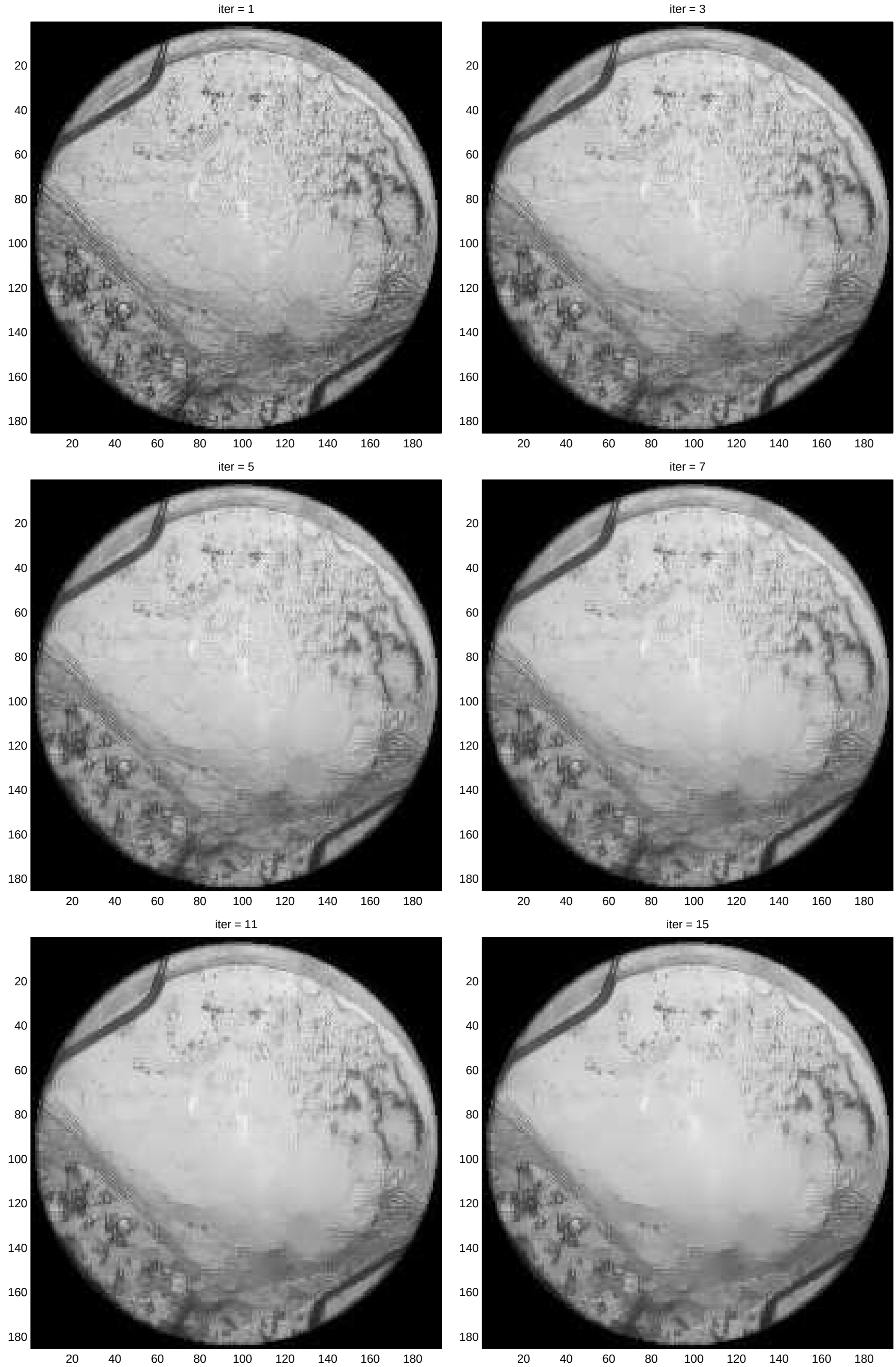

Figure 2: Filtering a PCO image with anisotropic diffusion

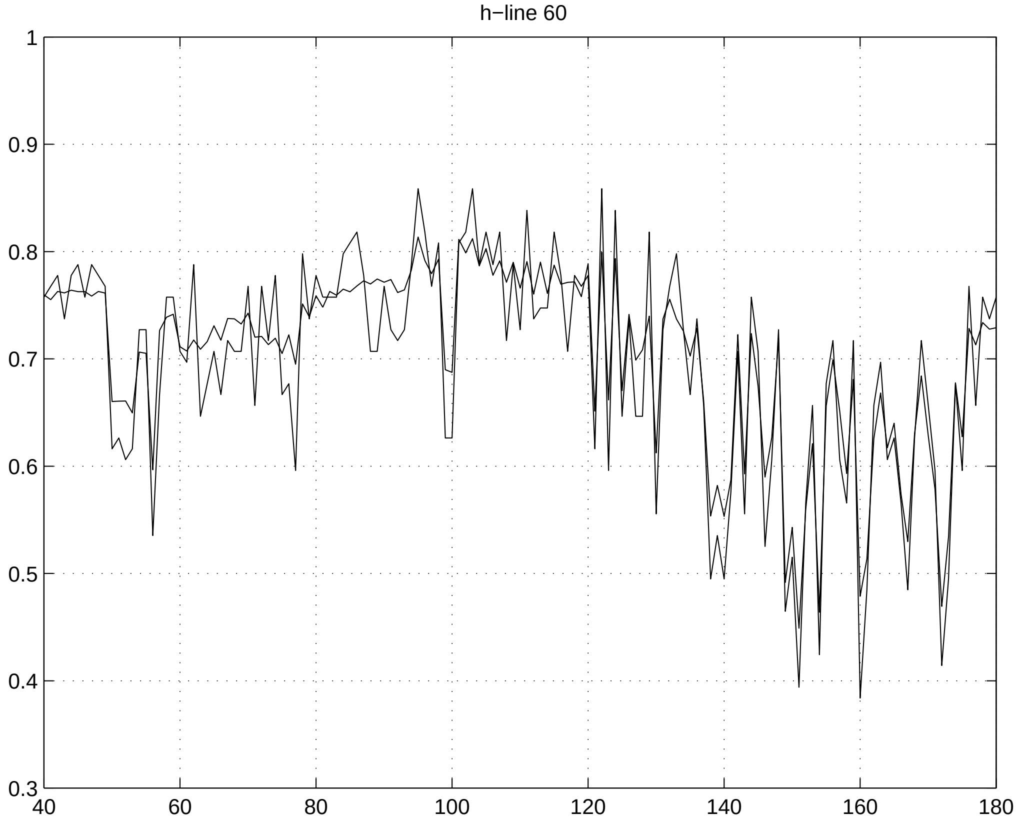

Figure 3: A selected horizontal line of the iterated image

References

[1] V. Caselles, J.-M. Morel, G. Sapiro, and A. Tannenbaum, "Introduction to the special issue on partial differential equations and geometry driven diffusion in image processing," *IEEE Transactions on Image Processing*, vol. 7, pp. 269–273, March 1998.

[2] G. H. Cottet and M. El Ayyadi, "A Voltera type model for image processing," *IEEE Transactions on Image Processing*, vol. 7, pp. 292–303, March 1998.

[3] S. T. Acton, "Multigrid anisotropic diffusion," *IEEE Transactions on Image Processing*, vol. 7, pp. 280–291, March 1998.

[4] J. Weickert and B. M. t. H. Romney, "Efficient and reliable schemes for nonlinear diffusion filtering," *IEEE Transactions on Image Processing*, vol. 7, pp. 399–410, March 1998.

[5] P. Perona and J. Malik, "Scale space and edge detection using anisotropic diffusion," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 12, pp. 629–630, 1990.

[6] A. P. Papliński and J. F. Boyce, "A first-order wave equation in modelling the behaviour of epithelial cells in an eye posterior capsule," in *Proceedings of the 6th IEEE International*

Workshop on Intelligent Signal Processing and Communication Systems (ISPACS'98), (Melbourne, Australia), November 1998.

[7] S. A. Barman, J. F. Boyce, D. J. Spalton, P. G. Ursell, and E. J. Hollick, "Measurement of posterior capsule opacification," in *Proceedings of the Conference on Medical Image Understanding and Analysis, MIUA97*, July 1997. Oxford, U.K.

[8] D. J. Spalton and P. G. Ursell, "Incidence of PCO with PMMA, Acrilic and Silicone IOLs: Two year follow-up," in *Symposium on Cataract, IOL and Refractive Surgery. Congress on Ophthalmic Practice Management*, June 1996. Seattle, Washington.

[9] A. P. Papliński and J. F. Boyce, "Segmentation of a class of ophthalmological images using a directional variance operator and co-occurrence arrays," *Optical Engineering*, vol. 36, pp. 3140–3147, November 1997.

[10] A. P. Papliński, "Directional filtering in edge detection," *IEEE Trans. Image Proc.*, vol. 7, pp. 611–615, April 1998.

[11] A. P. Papliński and J. F. Boyce, "Co-occurrence arrays and edge density in segmentation of a class of ophthalmological images," in *Proceedings of the 4rd Conference on Digital Image Computing: Techniques and Applications, DICTA97*, (Auckland,), pp. 521–528, December 1997.

[12] A. P. Papliński and J. F. Boyce, "Tri-directional filtering in processing a class of ophthalmological images," in *Proceedings of the IEEE Region 10 Annual Conference, TENCON'97*, (Brisbane), pp. 687–690, December 1997.

[13] MathWorks, *MATLAB Reference Guide*. The MathWorks Inc., 1997.