CSE460 Optimization and Constraint Solving

Assignment 1

Due 12 noon Friday, 1st of September, 2006

The purpose of this assignment is to demonstrate problem solving with linear and integer programming techniques and to practice basic modelling of such problems.

1. Consider the following production problem: A company produces four products (a, b, c, d). The production takes four different resource types into account (p, q, r, s). The company makes 1 unit of profit from each unit of b, c, and d produced and 10 units of profit from each unit of a. The available amounts of resources are $p : 2; q : 4; r : 22; s : 8$. The resource consumptions are:

	p	q	r	s
a	4	2	2	4
b	1	1	1	1
c	0	1	2	1
d	0	0	6	2

   (a) Write down the linear program to find the maximum profit production plan (the produced amounts can be fractional). **[1 mark]**

   (b) Give the optimal production plan. **[1 mark]**

   (c) Give the dual of your linear program from (a) and its solution. **[3 marks]**

   (d) Interpret the meaning of the dual problem variables. **[2 marks]**

   (e) Verify your interpretation from (d) by modifying the primal problem in a sensible way and re-solving the modified problem. Does your interpretation agree with the result? **[2 marks]**

   Note: You can use any linear solver that you want for this and the subsequent problems. As discussed in the lectures, possibilities are: Excel, Lingo, Matlab, Mathematica, a library like GLPK, etc. If you want to use Lingo, you need to obtain the student version from the Library (it comes with the Winston books). It is of course also permissible to solve the problems manually.

2. Consider the following game: Two players A and B compete in guessing each other's card. In each round both players are each dealt a single card from a deck containing only the three cards "King", "Queen" and "Ace". After looking at his own card player A must guess which card player B holds. After this B looks at his own card and announces his guess as to which card A holds. If a player guesses correctly, he wins $1 from the opponent. The roles of player A and $B$are not swapped between rounds. Answer the following questions and back up your answers with a mathematical analysis.

   (a) Is this game fair? What is its value? **[1 marks]**

   (b) Can either player improve his prospects by bluffing? (ignore any psychological implications of bluffing) **[4 marks]**

   (c) Give optimal strategies for both players. **[6 marks]**

3. Consider the following planning problem: A company wants to open a number of central warehouses to supply their stores with goods. The warehouse locations under consideration are Sydney (S), Melbourne (M), Albury (A) and Ballarat (B). Each warehouse has a fixed annual cost of running it and must be supplied from a *single* warehouse. The costs are A: 30; S: 25; M: 40; B: 30. There are a total of 6 stores ($a \dots f$) that need to be supplied. Each warehouse has a maximum capacity for the number of stores it can supply. These are: A: 1; S: 3; M: 2; B: 3. The annual cost of supplying a store from a particular warehouse is given by the following cost matrix:

	A	S	M	B
a	20	24	11	25
b	42	22	29	67
c	1	5	73	59
d	10	73	13	43
e	93	35	63	25
f	47	65	55	71

(a) Write down a linear integer program to find the minimum cost warehouse allocation. **[2 marks]**

(b) Solve this program: Which warehouses need to be opened, and which stores do they have to supply? What is the total annual cost of this allocation? **[3 marks]**

(c) If two warehouses are opened in the same state, the company can negotiate a better deal with their contractors. Opening two warehouses in New South Wales saves a total of 10 units, opening two warehouses in Victoria saves 20 units. Modify your program from (a) to take this additional condition into account. **[2 marks]**

(d) Report the same decisions as for the original problem in (b) for the modified problem under (c). Does the solution change? **[1 mark]**

4. A telecommunications company wants to connect a number of transmitter stations to a base station with fiber cable. The stations can either be connected directly to the base station or they can be connected to another station at which all the incoming signals are bundled, amplified together (using a single amplifier) and then retransmitted to other stations. The task is to determine a minimum cost wiring of the transmitter stations to the base station.

   Two different types of cable (with different costs) are available: Type 2 ($25/meter) can carry 25000 connections, type 3 ($200/meter) can be used for up to 300,000 simultaneous connections. Only a single cable can be used to connect two stations.

   The maximum number of simultaneously demanded connections at each transmitter station is known in advance: Station 1 needs up to 25,000 connections, Station 2 up to 40,000 connections, Station 3 up to 30,000 connections, Station 4 up to 100,000 connections, Station 5 up to 20,000 connections.

   Each station at which a connection originates needs to be equipped with an amplifier. Two amplifier types are available: Type 1 (cost $1,000,000) can process 30,000 simultaneous connections, type 2 can process 1,000,000 connections. Type 2 is very expensive: $10,000,000. Only one amplifier can be used per station and it is used for all outgoing signals together and amplifies them equally. Amp type 1 can transmit signals over 20 km, while type 2 is strong enough to transmit over 100 km.

   The required cable lengths between stations (in km) are given in the following distance matrix:

Cable Length	Base	Station 1	Station 2	Station 3	Station 4	Station 5
Base	0	23	12	44	6	100
Station 1	23	0	21	11	29	18
Station 2	12	21	0	17	2	11
Station 3	44	11	17	0	1	9
Station 4	6	29	2	1	0	88
Station 5	100	18	11	9	88	0

   Design a Lingo model to find the minimum cost wiring. Note that the above problem data is intentionally kept simple so that it fits in the restrictions of the Lingo student version. This version is limited in the number of constraints and the number of variables that it allows, so you will have to design a compact model! Your model should, however, be designed in such a way that it scales well and performs well with other (more complex) data.

   You are allowed to design a model for a different linear modelling package of your choice instead of using Lingo, but in this case you have to be prepared to demonstrate your solution to the lecturer.

   [12 marks]

Submission Instructions

The above exercises contribute 40% to your total CSE460 mark.

Submission will be electronic. You have to zip all submitted files into one directory and send them as a mail attachment to `bernd.meyer@infotech.monash.edu.au`. As documentation you can either submit plain text files, PDF files and Postscript files. All models and source code must be properly commented. Each submission will receive an electronic receipt by email. If you do not receive a receipt, you must assume that your submission has not been received.

The assignment is due **12 noon Friday, 1st of September, 2006**.

Assignments handed in after the due date will attract a late penalty of 10% per day unless special consideration applies or there has been prior agreement in writing from the lecturer. No submission will be accepted later than one week after the due date.