Cops and Robbers on Graphs

David Ellison

RMIT, School of Science

david.ellison2@rmit.edu.au

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Overview

Game of Cops and Robbers

- Given a graph G:
- The cop chooses his starting position on a vertex of G.
- The robber chooses his starting point.
- They move each in turn from one vertex to an adjacent vertex.
- They can see each other at all times.
- Can the cop catch the robber?

Known Properties: Dismantlability and Capture Time

Theorem (Characterisation of Copwin Graphs)

A graph is copwin if and only if it is dismantlable, i.e. if it can be reduced to a single vertex by successively removing vertices where the robber can be trapped. (Quilliot, 1978)

Theorem (Bounded Capture Time)

If G has n vertices, $n \geq 7$, then the capture time $ct(G)$ satisfies $ct(G) \leq n - 4$.(Gavenčiak, 2010)

The Impact of Loops

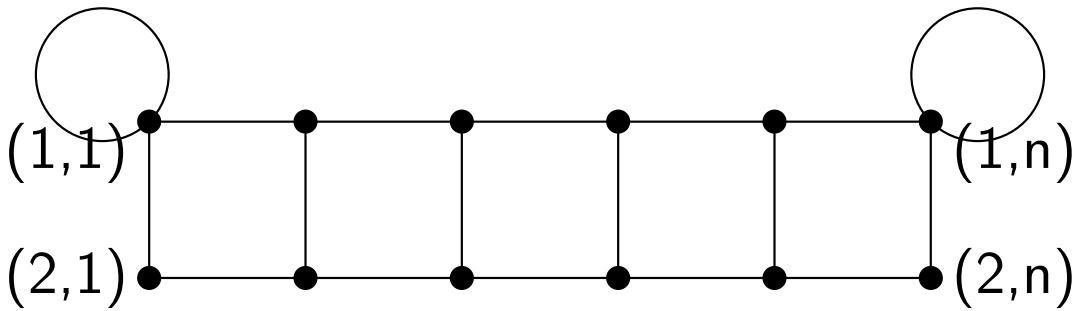


Figure: Partially looped $2 \times n$ grid

Cop moving away from the Robber

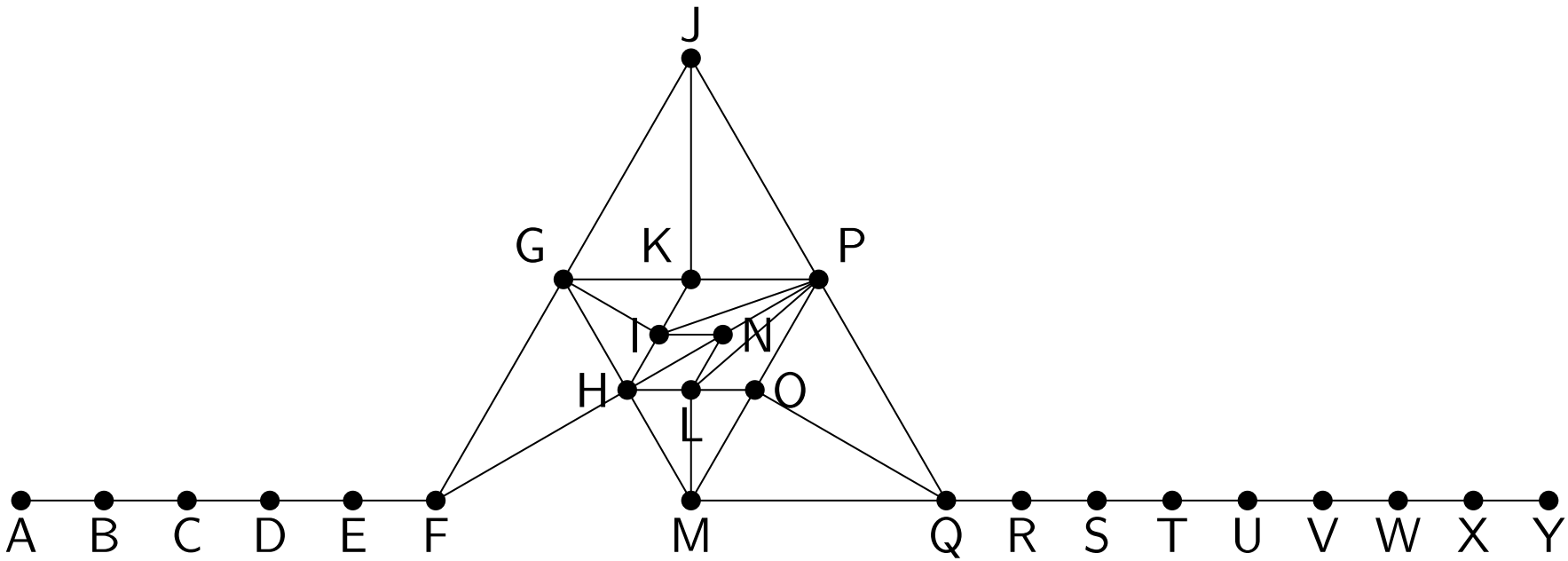


Figure: Graph G

Loops can help the Robber

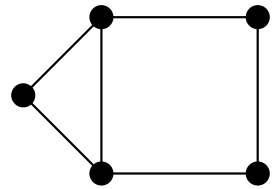


Figure: Graph H_1

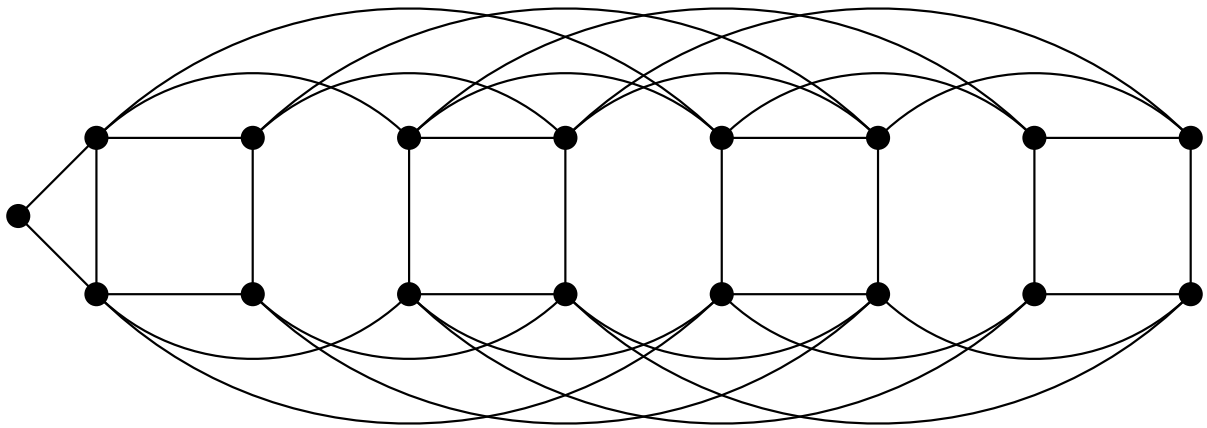
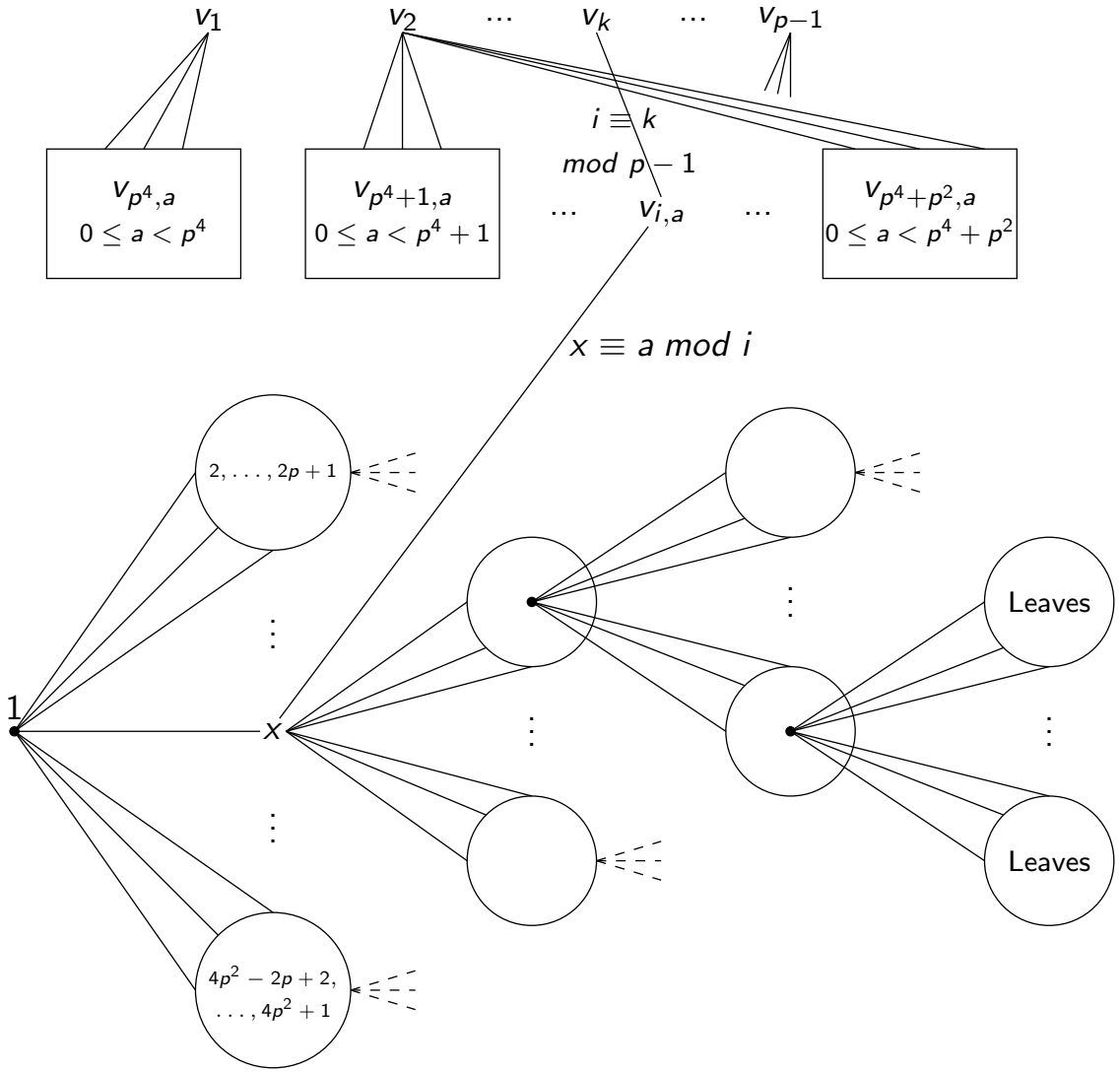


Figure: Graph H_2

Loops can also help the Cops



Cop Number and Loops

Given a graph G, let G^+ and G^- be the graphs obtained by adding or removing loops on every vertex respectively.

Proposition (Hahn et al.)

$c(G^+) \leq c(G^-) + 1$

Proposition

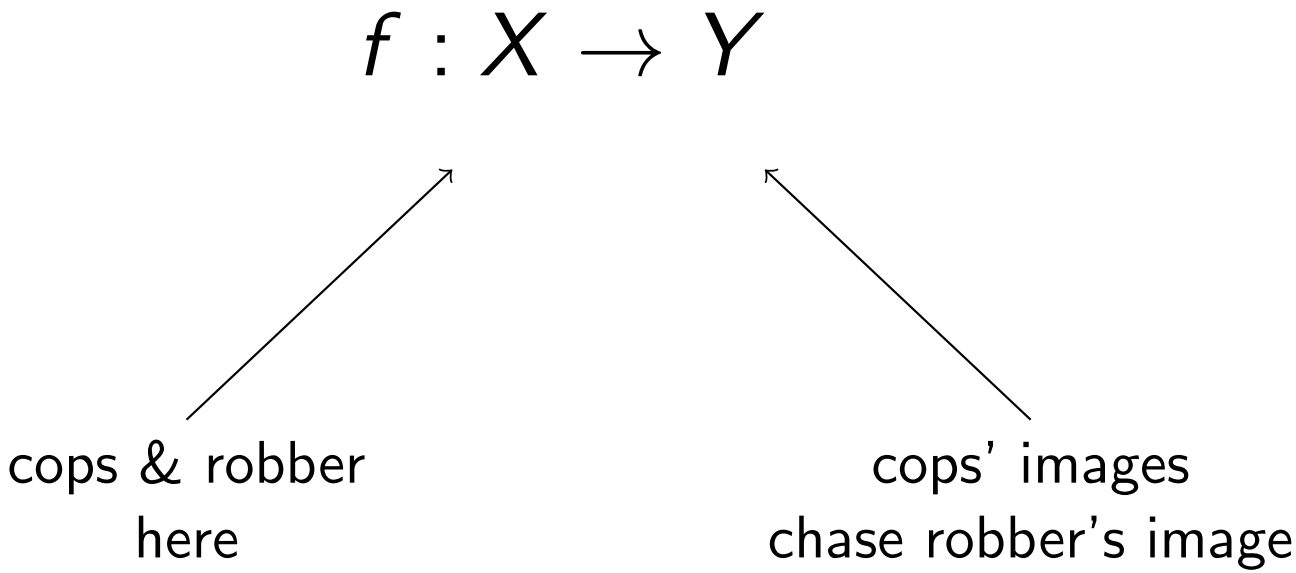
$c(G^-) \leq 2c(G^+)$

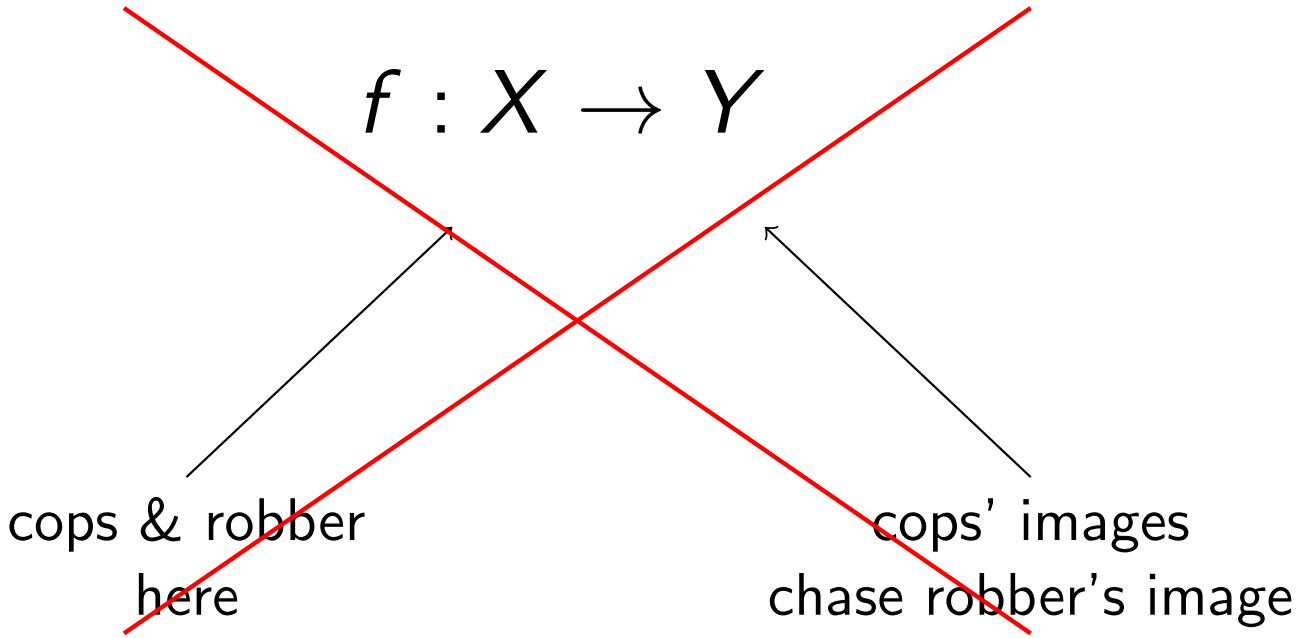
Proposition

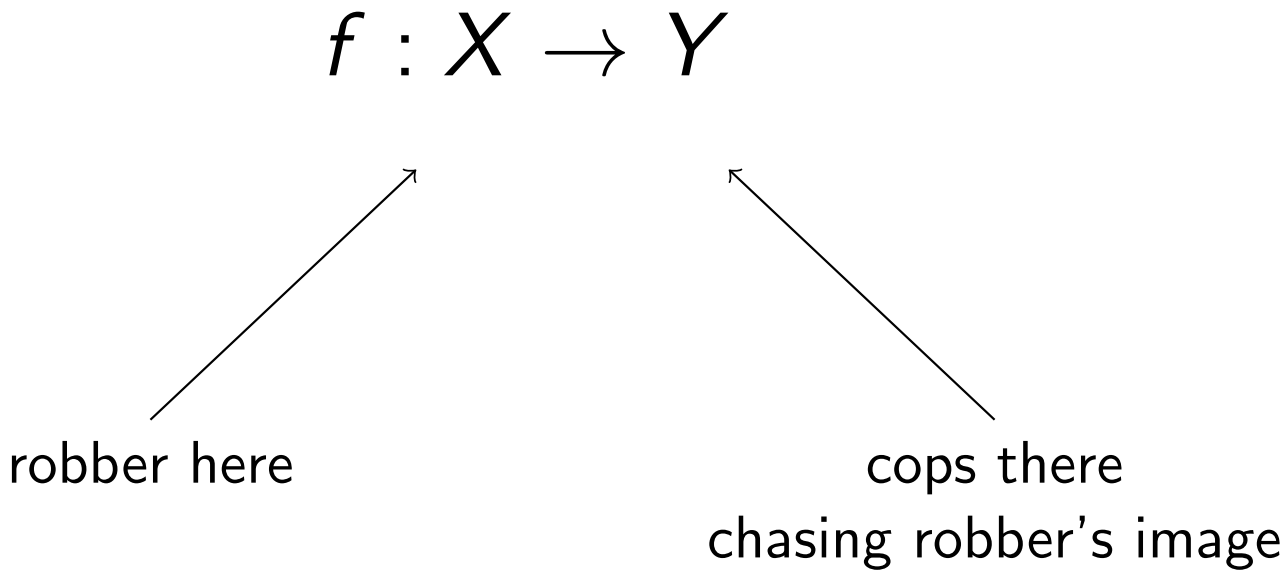
$\forall n, \exists G_n : c(G_n^+) = n \text{ and } c(G^-) = 2n - 1$

Conjecture

$c(G^+) < 2c(G^-)$


$f : X \to Y$
cops & robber
here
cops' images
chase robber's image


$f : X \to Y$
cops & robber
here
cops' images
chase robber's image


$f : X \to Y$
robber here
cops there
chasing robber's image

Theorem (Homotopy Invariance)

If two homomorphisms are homotopic, they have the same cop number and their capture times differ by the homotopic distance at most.

Theorem (Characterisation of Copwin Graphs)

A graph is copwin if and only if it is contractible.

Thank you for your attention!

No cops or robbers were harmed in the making of this presentation.