Can Hamiltonian Cycle Problem be Solved with High Probability in Polynomial Time?

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Definition

A Hamiltonian Cycle (HC for Short)

Given a graph **G**, a simple path that starts from an arbitrary node, visits all nodes exactly once and returns to the initial node is called a **Hamiltonian cycle** or a **tour**.

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- The HCP is an **NP-Complete** problem, that is, no-one has found an efficient solution algorithm with polynomial complexity time for it, yet.

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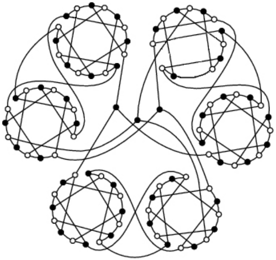

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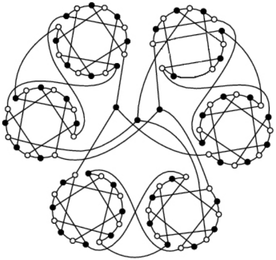

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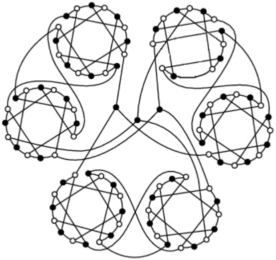

$$\frac{\#\text{ of HCs}}{\#\text{ of spanning subgraphs}} = \frac{76800}{3^{94}} = \mathcal{O}(10^{-40})$$

- Despite this, can a **random walk** be designed that has a "good chance" of finding a HC?

Notations

The HCP in this Talk

There is a directed graph, namely $\mathbf{G}$, on $\mathbf{n}$ nodes with no self-loops. Suppose $\mathcal{S} = \{1, 2, \ldots, n\}$ is the set of all nodes and $\mathscr{A}$ is the set of all arcs in this graph.

For each node i, we can define two subsets $\mathcal{A}(i) = \{a \in \mathcal{S} | (i, a) \in \mathscr{A}\}$ and $\mathcal{B}(i) = \{b \in \mathcal{S} | (b, i) \in \mathscr{A}\}$.

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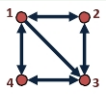
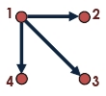
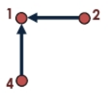
Example







$\mathcal{S} = \{1, 2, 3, 4\}$ $\quad$ $\mathcal{A}(1) = \{2, 3, 4\}$ $\quad$ $\mathcal{B}(1) = \{2, 4\}$

Embedding in Markov Decision Processes

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- They converted the deterministic HCP to a particular **average-reward Markov decision process**.
- In 2000, Feinberg converted the HCP to a class of Markov decision processes, the so-called **weighted discounted Markov decision processes**.
- MDP embedding implies that you can search for a Hamiltonian cycle in a nicely structured **polyhedral domain of discounted occupational measures**.

Domain of Discounted Occupational Measures

$\mathcal{H}_\beta$ Polytope Associated with the Graph G; $\beta \in (0,1)$

$$\begin{aligned}
\sum_{a\in\mathcal{A}(1)} x_{1a} - \beta \sum_{b\in\mathcal{B}(1)} x_{b1} &= 1-\beta^n \\
\sum_{a\in\mathcal{A}(i)} x_{ia} - \beta \sum_{b\in\mathcal{B}(i)} x_{bi} &= 0 \ ; \ i = 2,3,\ldots,n \\
\sum_{a\in\mathcal{A}(1)} x_{1a} &= 1 \\
x_{ia} \geq 0 \quad &; \quad \forall\, i \in \mathcal{S} \ , \ a \in \mathcal{A}(i)
\end{aligned}$$

Hamiltonian Extreme Points

Theorem (Feinberg, 2000)

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If $\mathring{\mathbf{x}}$ is a Hamiltonian extreme point, then for each $i \in \mathcal{S}$, $\exists!\ a \in \mathcal{A}(i)$ so that, $\mathring{x}_{ia} > 0$. These positive variables trace out a tour in the graph.

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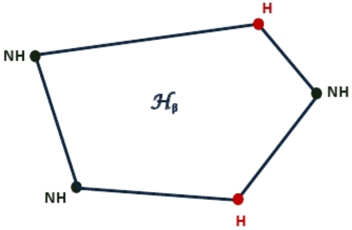

Illustration

Example

$$
\begin{aligned}
x_{12}+x_{13}+x_{14}-\beta x_{21}-\beta x_{41} &= 1-\beta^4 \\
x_{21}+x_{23}-\beta x_{12}-\beta x_{32} &= 0 \\
x_{32}+x_{34}-\beta x_{13}-\beta x_{23}-\beta x_{43} &= 0 \\
x_{41}+x_{43}-\beta x_{14}-\beta x_{34} &= 0 \\
x_{12}+x_{13}+x_{14} &= 1 \\
x_{ia} &\geq 0 \ ; \ i=1,2,3,4 \ , \ a\in\mathcal{A}(i)
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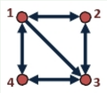

Illustration (Cont.)

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- **One particular feasible solution**:

$$x_{12} = 1\,,\ x_{23} = \beta\,,\ x_{34} = \beta^2\,,\ x_{41} = \beta^3\,,$$

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- **One particular feasible solution**:

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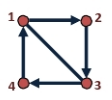
- It traces out the **standard Hamiltonian cycle** $1 \to 2 \to 3 \to 4 \to 1$.



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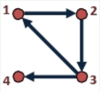
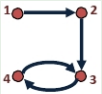


A short cycle



A noose cycle

Hamiltonian and non-Hamiltonian Extreme Points of $\mathcal{H}_\beta$

Theorem (Ejov et. al., 2009)

Consider a graph G and the corresponding polytope $\mathcal{H}_\beta$. Any extreme point **x** *identifies either a* **Hamiltonian cycle** *or a 1-randomized policy that traces out a path in G that is a* **combination of a short cycle and a noose cycle**.

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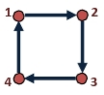
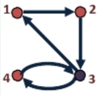


A Hamiltonian extreme point



A combined extreme point

non-Hamiltonian Extreme Points [Eshragh and Filar, 2011]

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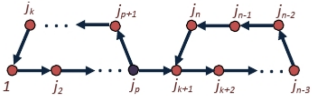

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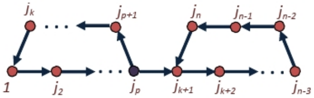
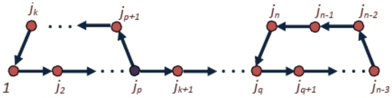


2. Type II



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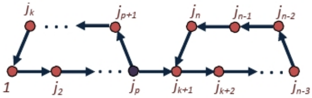

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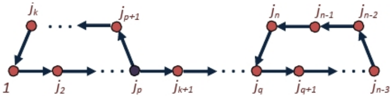
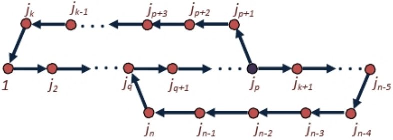


3. Type III



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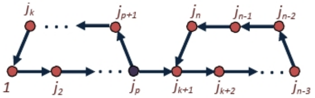

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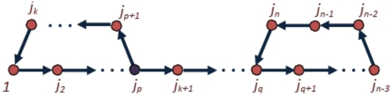

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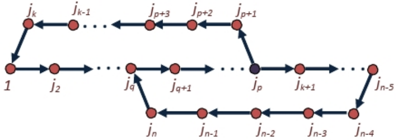

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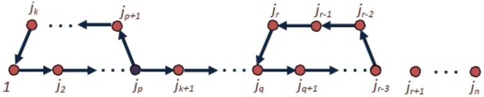

The Prevalence of Hamiltonian Extreme Points

- What is the **Ratio** of the number of Hamiltonian extreme points over the number of non-Hamiltonian ones Type I, II, III and IV?

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- We utilized **Erdös-Rényi Random Graphs $G_{n,p}$**.

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Theorem (Eshragh and Filar, 2011)

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3. $$\frac{E[\#\text{ of Hamiltonian Extreme Points}]}{E[\#\text{ of NH Extreme Points Type IV}]} = \mathcal{O}(e^{-n})$$

The Generic Structure of Hamiltonian Extreme Points

Lemma (Eshragh et. al., 2009)

If $\mathring{\mathbf{x}}$ is a Hamiltonian extreme point corresponding to tour τ in the given graph G, then its components must be as follows:

$$\mathring{x}_{ia} = \begin{cases} \beta^{k-1} & ; \text{ if } (i,a) \text{ is the } k^{th} \text{ arc in tour } \tau \text{ starting from node 1} \\ 0 & ; \text{ otherwise} \end{cases}$$

Reducing the Feasible Region

The Wedged Hamiltonian Polytope $\mathcal{WH}_\beta$ [Eshragh et. al. 2009]

$$\mathcal{H}_\beta$$

and

$$\beta^{n-1} \le \sum_{a \in \mathcal{A}(i)} x_{ia} \le \beta \quad \text{for } i = 2, 3, \ldots, n$$

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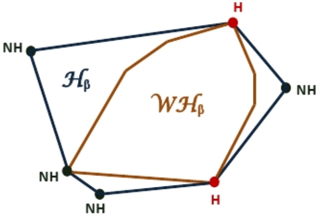

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Consider the graph G and polytopes $\mathcal{H}_\beta$ and $\mathcal{WH}_\beta$. For $\beta \in \left((\mathbf{1} - \frac{\mathbf{1}}{\mathbf{n-2}})^{\frac{\mathbf{1}}{\mathbf{n-2}}}, \mathbf{1} \right)$, the intersection of extreme points of these two polytopes can be partitioned into two disjoint (possibly empty) subsets:

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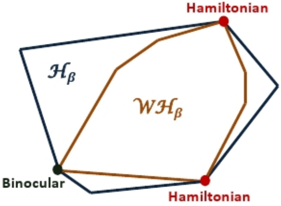
(i) **Hamiltonian extreme points**;

(ii) **binocular extreme points**.

Investigating HCs through Random Walks



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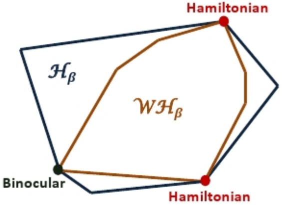

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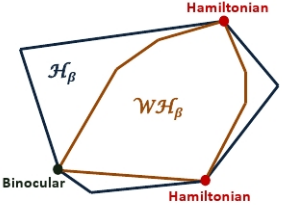

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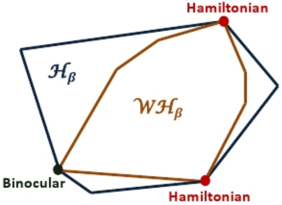

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2. Uniformly, choose one of the adjacent extreme points, at random and move to that one;
3. If the current extreme point is Hamiltonian, then **STOP** and claim the graph is Hamiltonian, otherwise, return to Step 2.

Results on Several Graphs

Graph	Iteration Number
Hamiltonian Graph on 6 Nodes	1
Hamiltonian Graph on 10 Nodes	1
Hamiltonian Graph on 20 Nodes	10
Hamiltonian Graph on 30 Nodes	12
Hamiltonian Graph on 40 Nodes	10
Hamiltonian Graph on 50 Nodes	2
Hamiltonian Graph on 60 Nodes	27
Hamiltonian Graph on 80 Nodes	11
Hamiltonian Graph on 100 Nodes	29
Hamiltonian Graph on 150 Nodes	34
Hamiltonian Graph on 200 Nodes	37
Hamiltonian Graph on 400 Nodes	52
Hamiltonian Graph on 800 Nodes	67

Further Developments

Conjecture (Eshragh and Filar, 2011)

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(i) *There exists a* **polynomial-time algorithm** *to generate extreme points of the polytope $\mathcal{WH}_\beta$, uniformly, at random.*

(ii) *For large values of β, the proportion of Hamiltonian extreme points in the the polytope $\mathcal{WH}_\beta$ is bounded below by $\frac{1}{\rho(n)}$, where $\rho(n)$ is a polynomial function of n.*

Exploiting the Conjecture

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- For a given Hamiltonian graph G,

$$\mathbf{Pr(\text{Required number of iterations} > \tau)} \leq \mathbf{e}^{-\frac{\tau}{\rho(n)}};$$

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A New Algorithm for the HCP

1. Construct the polytope $\mathcal{WH}_\beta$ corresponding to a given graph G, set β large enough and $t = 1$;
2. Generate an extreme point of polytope $\mathcal{WH}_\beta$, say $\mathbf{x}_t$, uniformly, at random. If $\mathbf{x}_t$ is a Hamiltonian extreme point, then **STOP** and claim that G is Hamiltonian;
3. If $t > \alpha\rho(n)$, then **STOP** and claim that **with high probability**, G is non-Hamiltonian. Otherwise, set $t = t + 1$ and return to Step 2.

- For a given Hamiltonian graph G,

$$\mathbf{Pr(Required\ number\ of\ iterations} > \boldsymbol{\tau}) \leq \mathbf{e}^{-\frac{\boldsymbol{\tau}}{\boldsymbol{\rho}(\mathbf{n})}};$$

- For a given graph G, we can solve the HCP, **with high probability**, in polynomial time.

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- The current extreme point has been sampled **uniformly**.

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- In all numerical examples we have tested so far, the graph of polytopes $\mathcal{WH}_\beta$ seemed to be regular.

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Theorem (Ye, 2011)

The **Simplex method** *can solve discounted Markov decision processes with a fixed discount factor in* **polynomial time**.

End

Thank you $\cdots$ Questions?