The Maximum Induced Planar Subgraph problem

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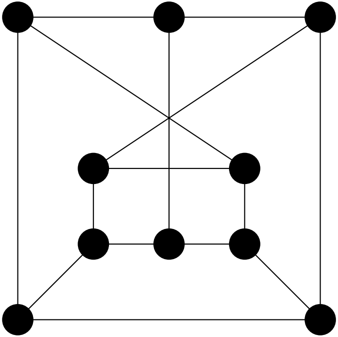
Joint work with Keith Edwards (Dundee) and Kerri Morgan

The problem

MAXIMUM *INDUCED* PLANAR SUBGRAPH (MIPS)
Input: Graph G
Output: set $P \subseteq V(G)$ such that

- the *induced* subgraph $\langle P \rangle$ is planar,
- $|P|$ is maximum.



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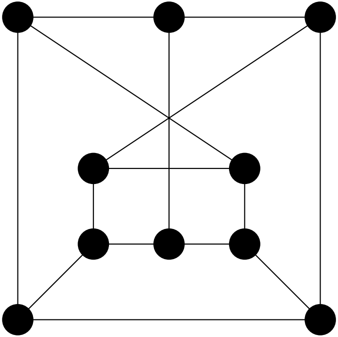

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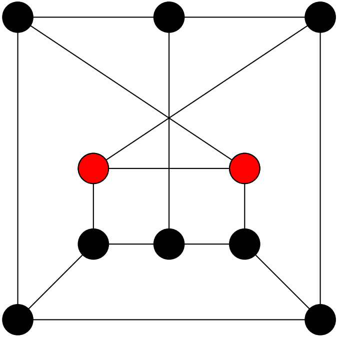

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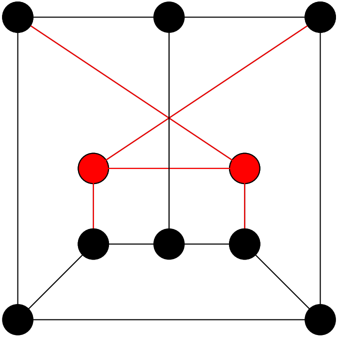

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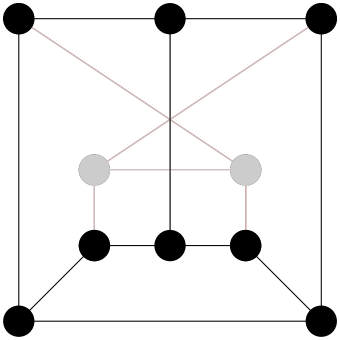

Complexity of MIPS

MIPS is

- NP-hard to solve exactly (Krishnamoorthy & Deo, 1979; Lewis & Yannakakis, 1980)
- also hard to approximate (Lund & Yannakakis, 1993): $\exists \varepsilon > 0$: cannot get performace ratio $n^{-\varepsilon}$ unless P = NP.
- approximable with performance ratio $\Omega(n^{-1}(\log n / \log \log n)^2)$ (Halldórsson, 2000)

Bounded degree MIPS

"Real" graphs have low degrees.

Approximation algorithms: max degree $\leq d$:

- Halldórsson & Lau, 1997:
  proportion of vertices included:
  $$\frac{1}{\lceil (d+1)/3 \rceil}$$
  - linear time
  - subgraphs found have max degree ≤ 2
- Edwards & Farr, *GD* 2001:
  proportion of vertices included:
  $$\frac{3}{d+1}$$
  - time $O(mn)$
  - subgraphs found are series-parallel

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  - induced outerplanar subgraph
- **Average degree $\leq d$: vertex removal algorithm**
- Experiments
- Connection with fragmentability
- Future work

Finding an Independent Set (classical heuristic)

Input: $G = (V, E)$
$P := \emptyset, \qquad R := V$
Loop: if $v \in R$ has degree ≤ 0 in P, move it to P.
Output: P

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Count $E(P, R)$ from each side:

$$\begin{aligned} d|P| &\geq |R| \\ d|P| &\geq n - |P| \\ (d+1)|P| &\geq n \\ |P| &\geq \frac{n}{d+1} \end{aligned}$$

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Proportion: $\dfrac{1}{d+1}$ (Turán)

Finding an Induced Forest

Input: $G = (V, E)$

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Loop: if $v \in R$ has degree ≤ 1 in P,
move it to P.

Output: P

Finding an Induced Forest

Input: $G = (V, E)$

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Loop: if $v \in R$ has degree ≤ 1 in the non-null portion P_1 of P, move it to P.

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Loop: if $v \in R$ has degree ≤ 1 in the non-null portion P_1 of P, move it to P.

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Stops when every vertex in R has degree ≥ 2 in P_1.

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Stops when every vertex in R has degree ≥ 2 in P_1.
$P_0 :=$ {isolated vertices in $\langle P \rangle$}; $\qquad P_1 = P \setminus P_0$.
Count $E(P_1, R)$ from each side:

$$\begin{aligned}
(d-1)|P_1| &\geq 2|R| \\
(d-1)|P_1| &\geq 2n - 2|P| \\
(d+1)|P_1| + 2|P_0| &\geq 2n \\
|P| = |P_1| + |P_0| &\geq \frac{2n}{d+1}
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Proportion: $\dfrac{2}{d+1}$ (Alon, Mubayi, Thomas, 2001)

Finding an Induced Planar Subgraph

Algorithm from Edwards & Farr, 2002 (outline):

Input: $G = (V, E)$

$P := \emptyset$, $R := V$

$P_0 :=$ forest portion of $\langle P \rangle$; $P_1 = P \setminus P_0$.

Loop: if $v \in R$ has $d_{P_1}(v) \leq 2$,

can *either* move it to P (increases $|P|$)

or swap it with an appropriate vertex in P.

(This swap *may* require $d_{P_0}(v) \leq 1$.)

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Count $E(P, R)$, or $E(P_1, R)$, from each side.

Obtain: Proportion: $\dfrac{3}{d+1}$

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Finds an induced series-parallel subgraph.

Finding an Induced Outerplanar Subgraph

Algorithm from Morgan & Farr, 2007 (outline):

Input: $G = (V, E)$

$P := \emptyset$, $R := V$

$P_1 :=$ union of components of size ≥ 3 of $\langle P \rangle$.

Firstly: $P :=$ maximal induced forest of G,
then make some easy additions to P.

Loop: if $v \in R$ has degree ≤ 2 in P_1,
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When stopped, every vertex in R has degree ≥ 3 in P_1.

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When stopped, every vertex in R has degree ≥ 3 in P_1.
Count $E(P_1, R)$ from each side.
Obtain: Proportion: $\dfrac{3}{d + 5/3}$

Proportion of vertices removed

Max degree $\leq d$:

$$\text{proportion} \quad \leq \frac{d-2}{d+1}$$

E & F 2001,2002

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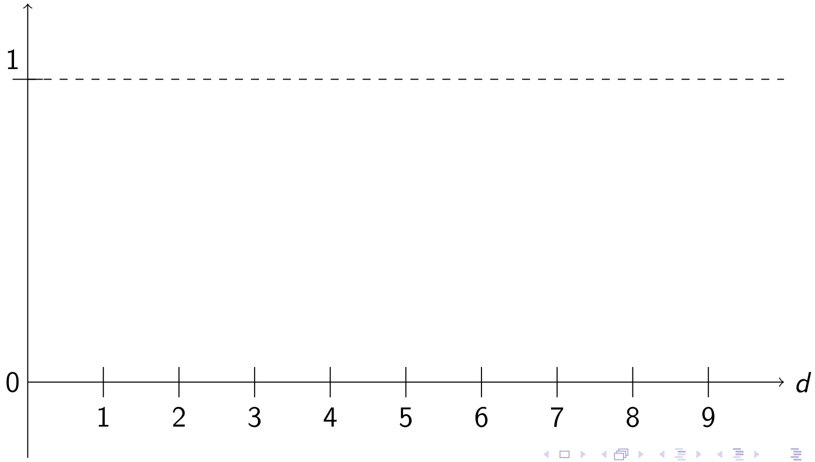

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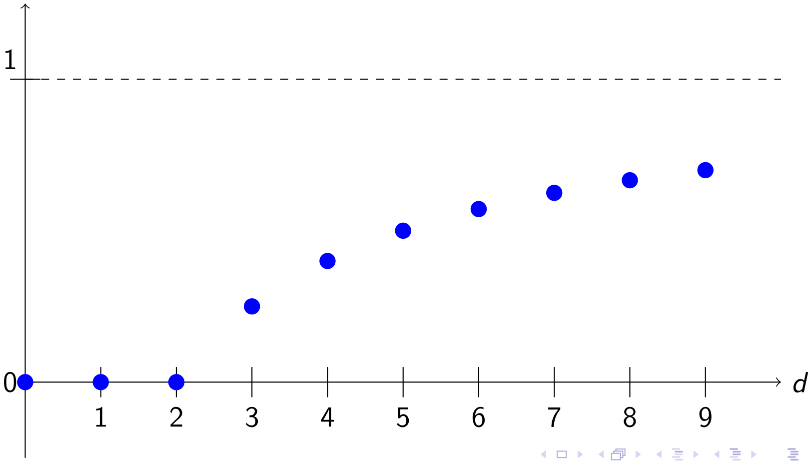

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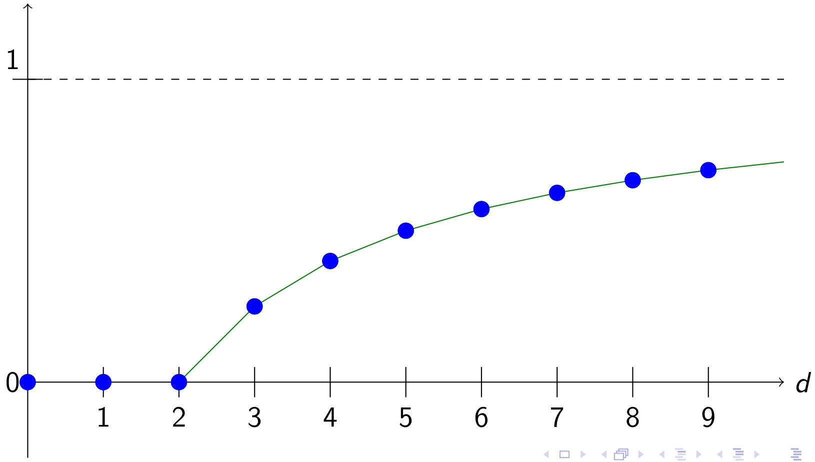

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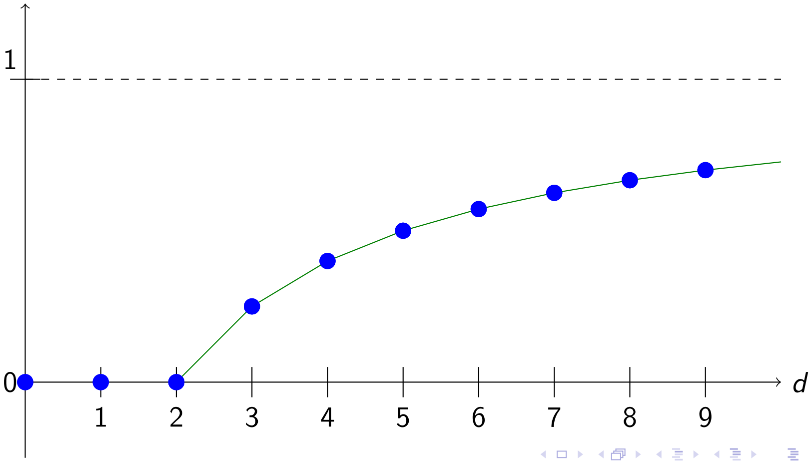
E & F 2001,2002 2003,2007



Series-parallel reductions

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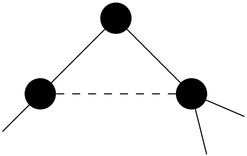
3. degree 2: 

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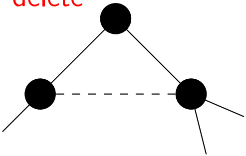

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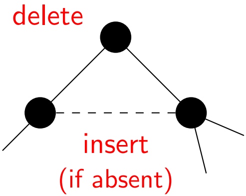
3. degree 2: 

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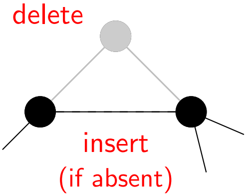

G ⟶ do series-parallel reductions for as long as possible ⟶ $r(G)$

reduced graph

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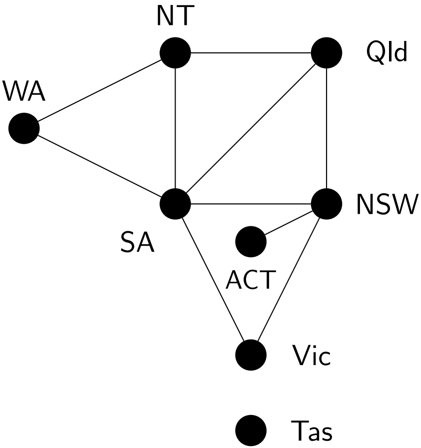

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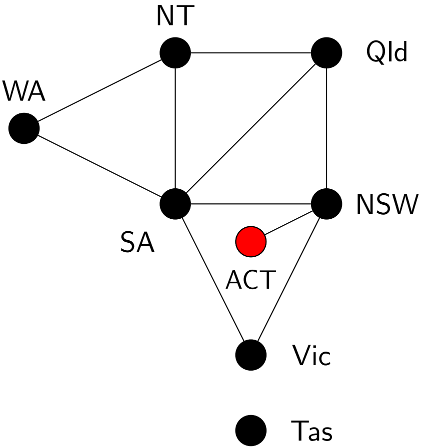

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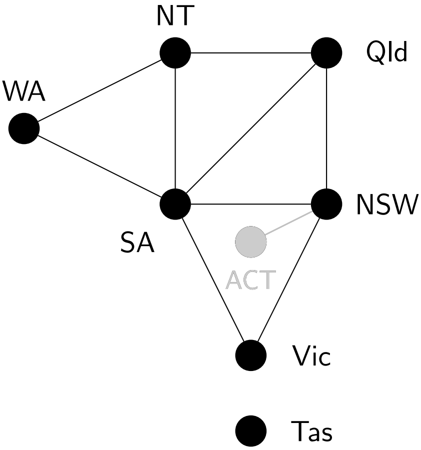

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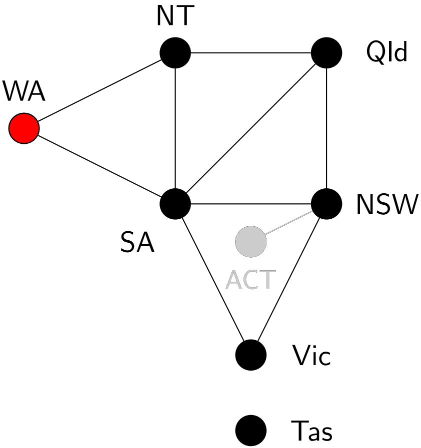

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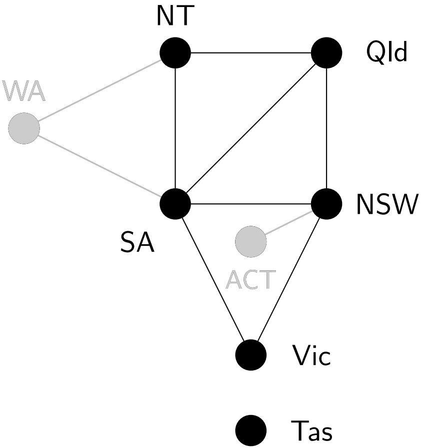

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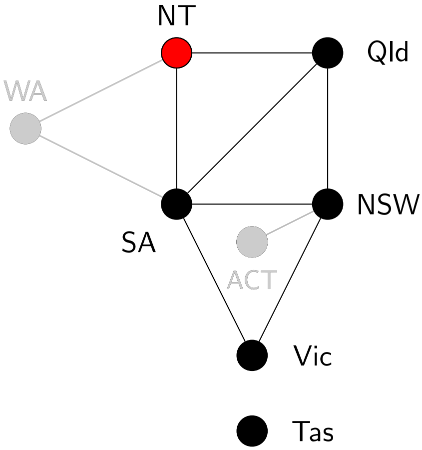

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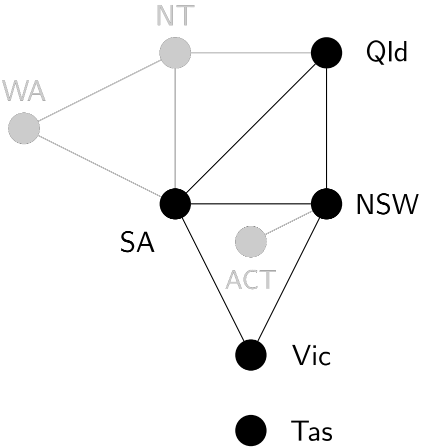

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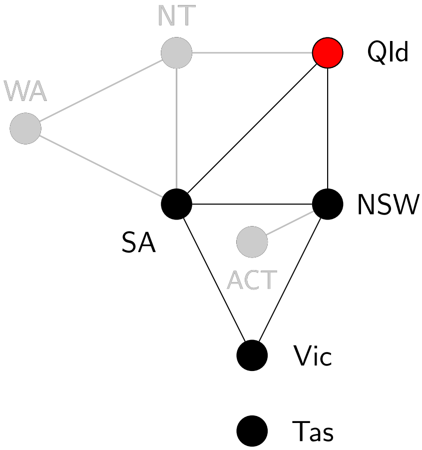

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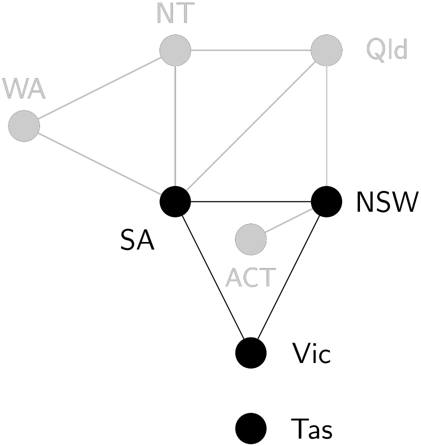

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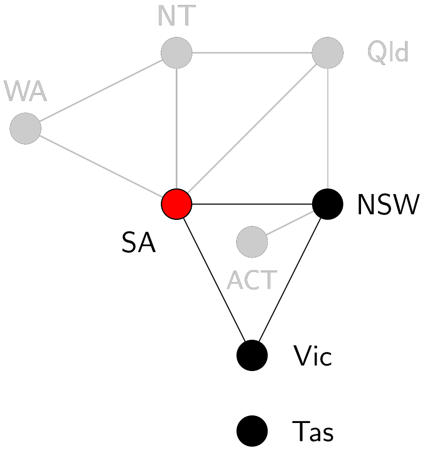

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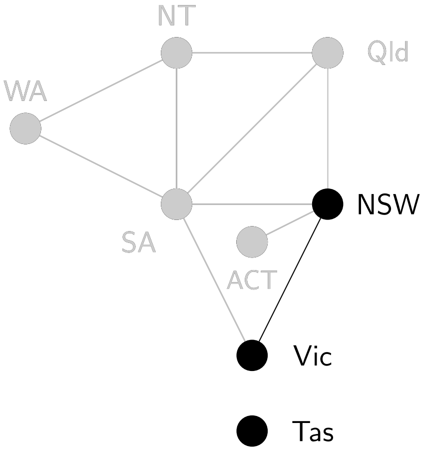

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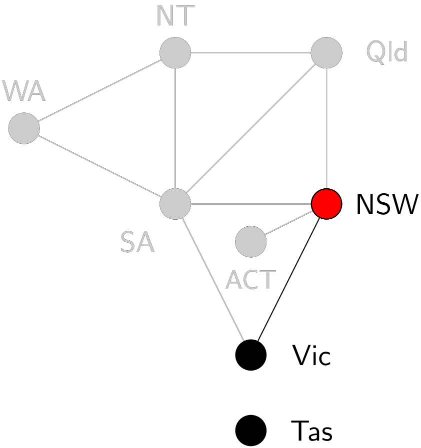

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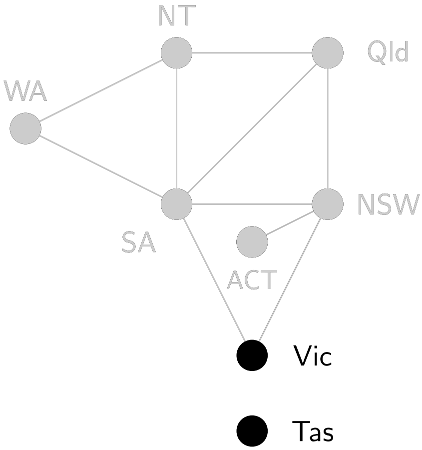

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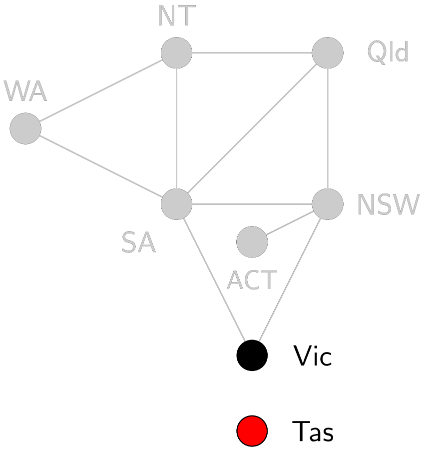

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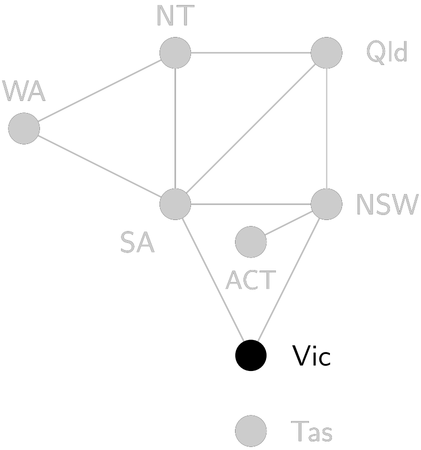

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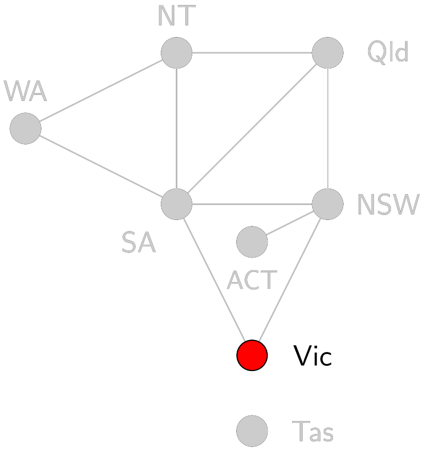

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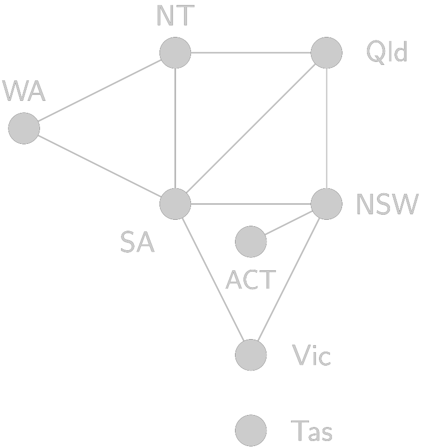
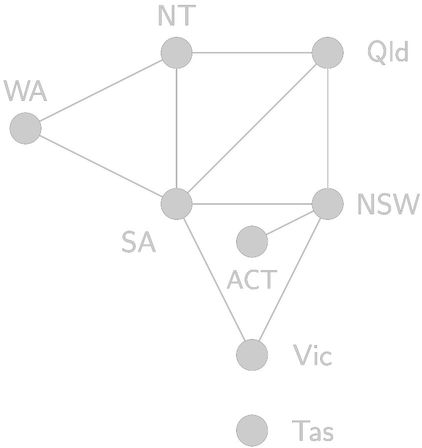


G ⟶ [do series-parallel reductions for as long as possible] ⟶ $r(G)$

reduced graph

Example:

G :



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Fact

G is series-parallel $\Leftrightarrow$ $r(G)$ is empty.

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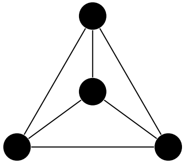
Theorem (Duffin, 1965)

G is series-parallel $\Leftrightarrow$ G contains no subdivision of K_4.



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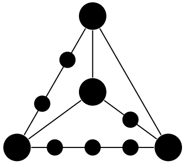

Algorithm 1.

1. **Input:** Graph G.
2. $P := V(G)$ // *vertices to be kept*

   $R := \emptyset$ // *vertices to be removed*

   $$\rho := \left\lfloor \sum_{v \in V(r(G))} \frac{d_{r(G)}(v) - 2}{d_{r(G)}(v) + 1} \right\rfloor$$

3. **while** ($|R| < \rho$ and $r(\langle P \rangle)$ is nonempty)

   {

   $w :=$ vertex in P with maximum degree in $r(\langle P \rangle)$

   $P := P \setminus \{w\}$

   $R := R \cup \{w\}$

   }

4. **Output:** $\langle P \rangle$.

Theorem (E & F, 2003, 2007)

If G has min degree ≥ 3, then Algorithm 1 finds a series-parallel subgraph of G, and the number $|R(G)|$ of vertices removed satisfies

$$|R(G)| \leq \sum_{v \in V(G)} \frac{d(v) - 2}{d(v) + 1}.$$

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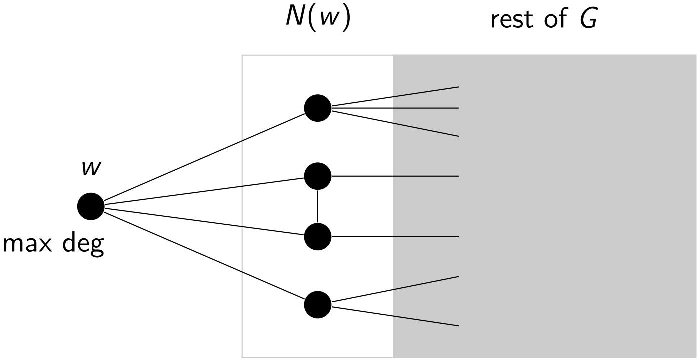
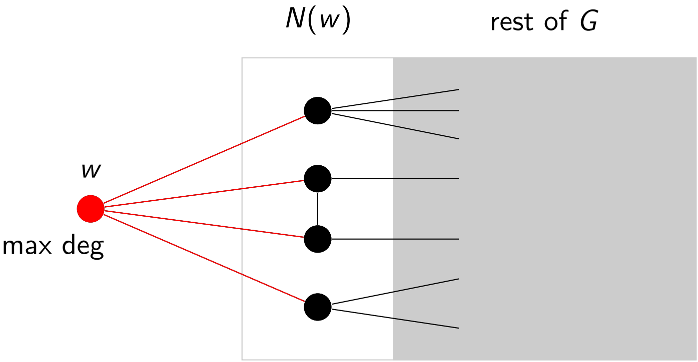
Proof. Induction on n.

Inductive basis: empty graph
(min degree ≥ 3: no vertices of degree 0,1,2).

Now let G be any graph with min degree ≥ 3 ...


N(w)
rest of G
w
max deg



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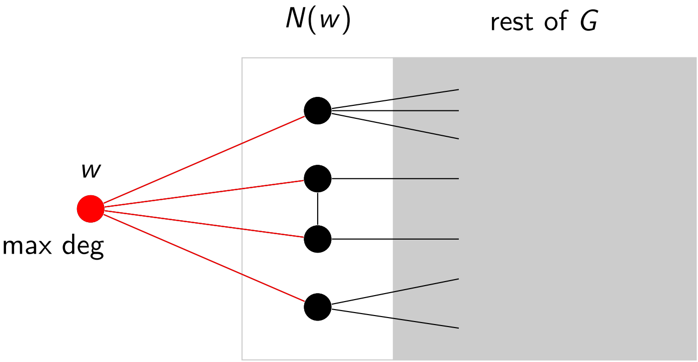
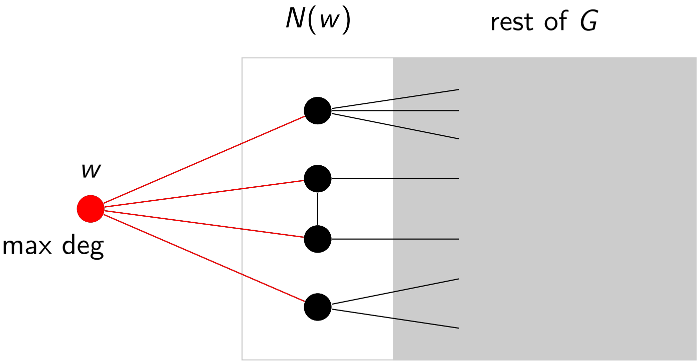
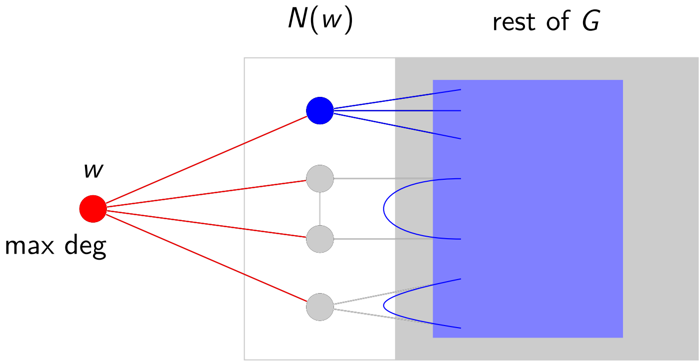


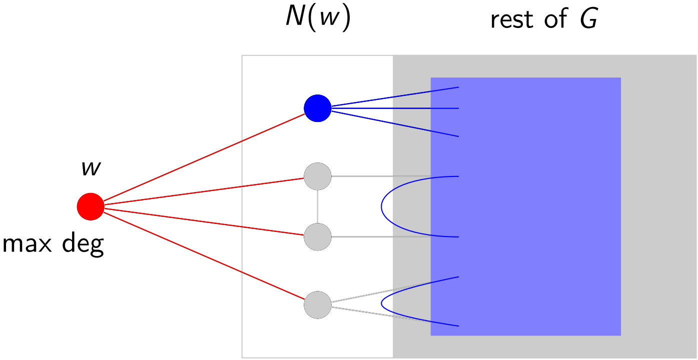
$|R(G)| \leq \quad 1 \; +$



$$|R(G)| \leq \quad 1 \ + \quad |R(\ G - w \ )|$$

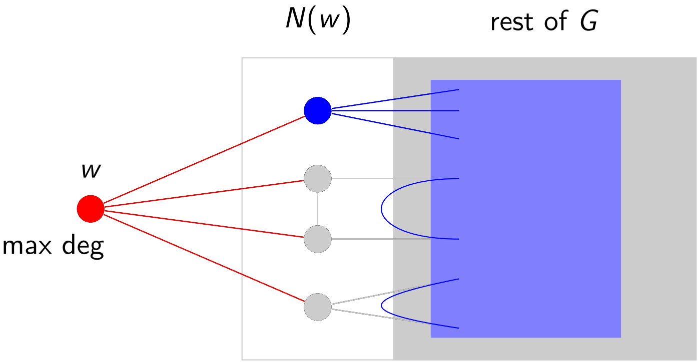


$$|R(G)| \leq 1 + |R(r(G-w))|$$



$|R(G)| \leq \quad 1 \; + \quad |R({\color{blue}r}(G-w))|$

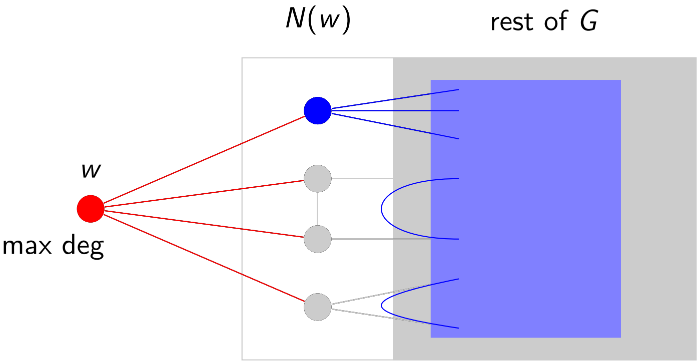
induction:



$$|R(G)| \le 1 + \underbrace{|R({\color{blue}r}(G-w))|}_{\le}$$

induction:

$$\le \sum_{v \in V({\color{blue}r}(G-w))} \frac{d_{{\color{blue}r}(G-w)}(v) - 2}{d_{{\color{blue}r}(G-w)}(v) + 1}$$



$$|R(G)| \leq 1 + \underbrace{|R(r(G-w))|}_{\underbrace{\leq}_{\sum_{v \in V(r(G-w))} \frac{d_{r(G-w)}(v)-2}{d_{r(G-w)}(v)+1}}}$$

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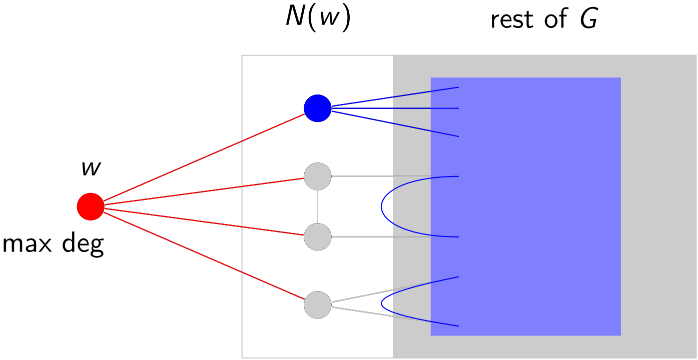

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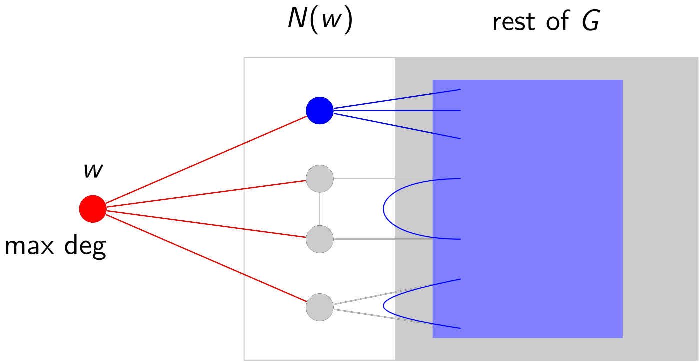

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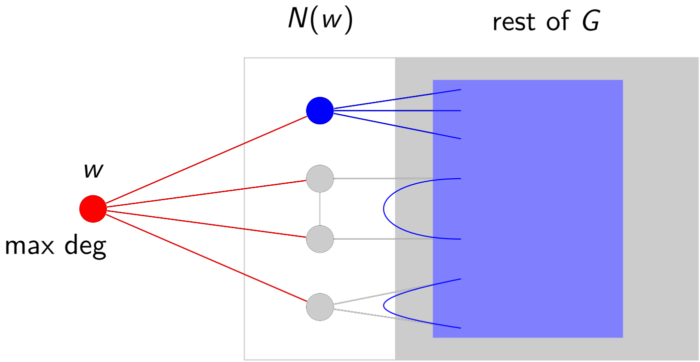

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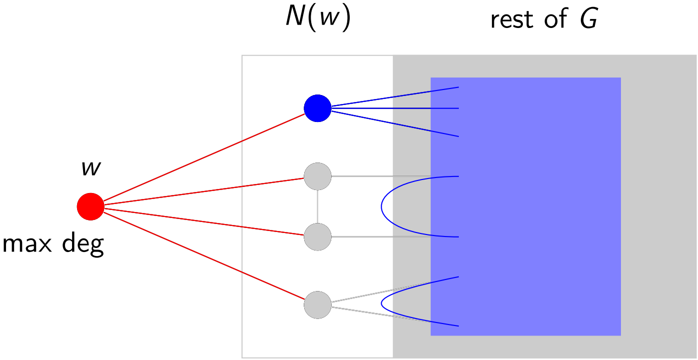

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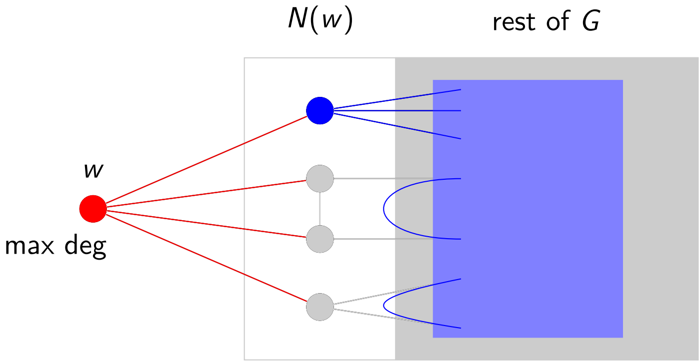

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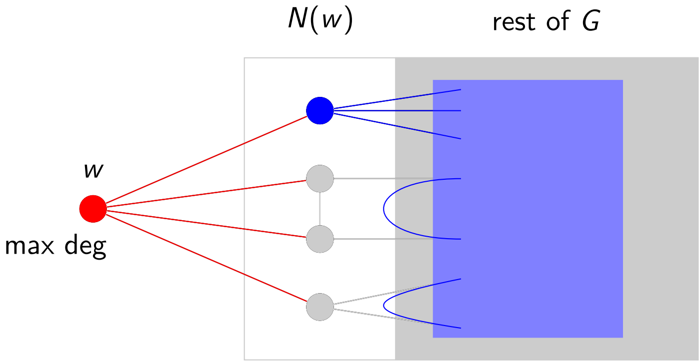

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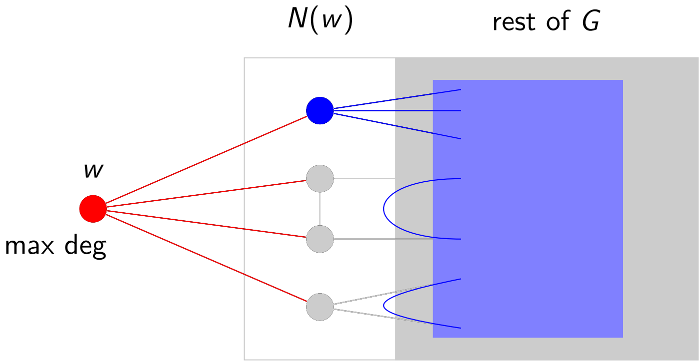

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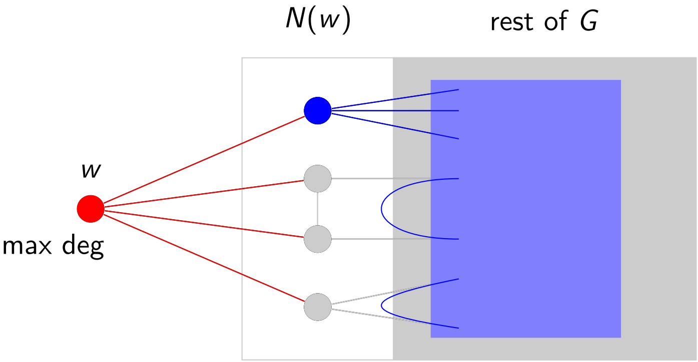

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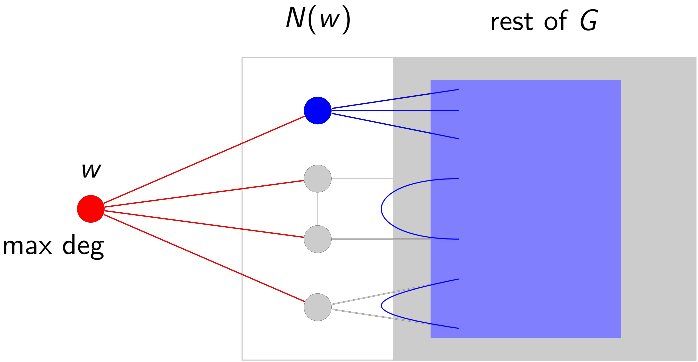

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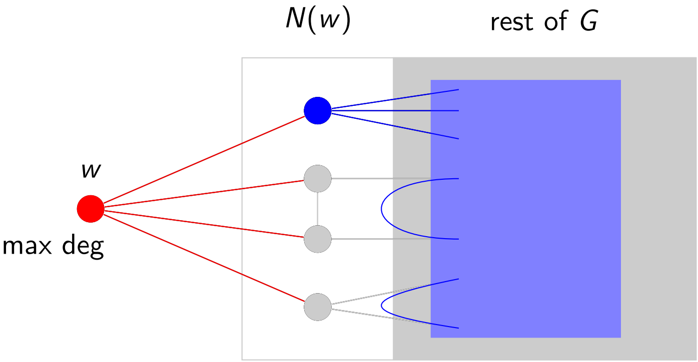

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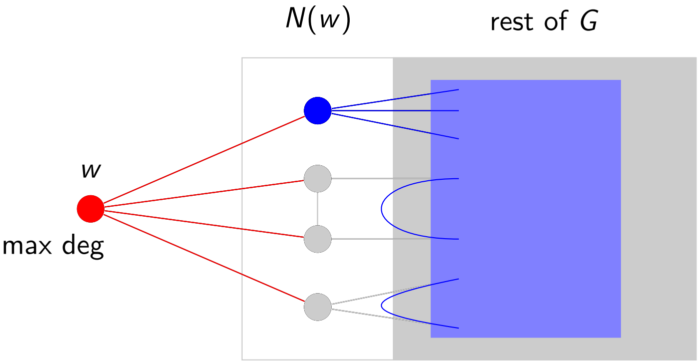

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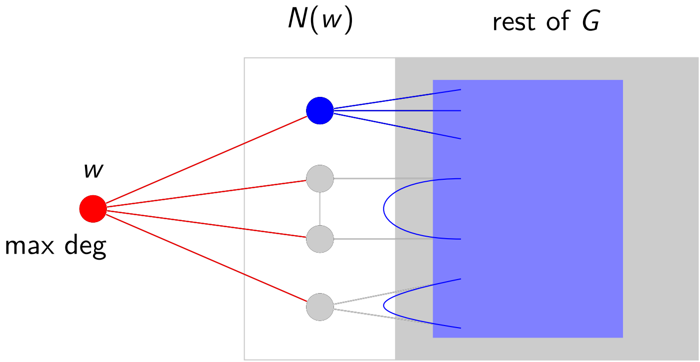

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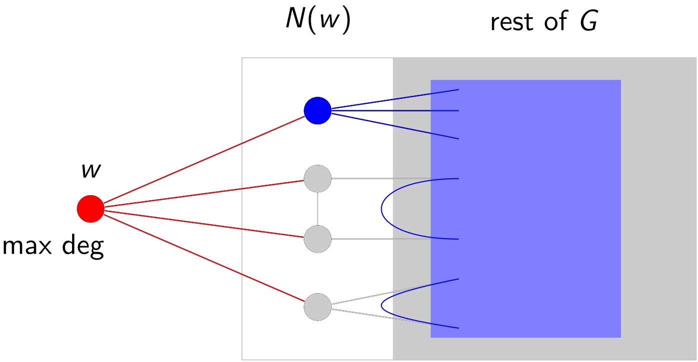

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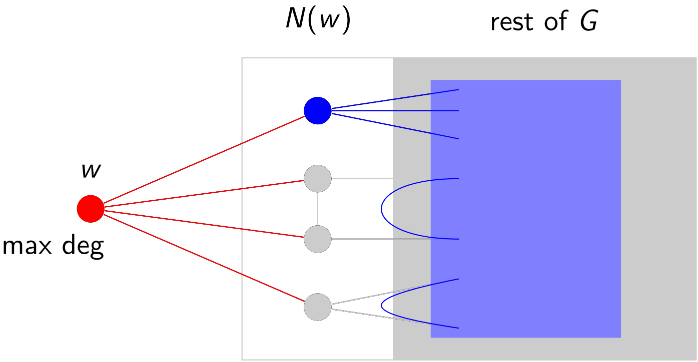

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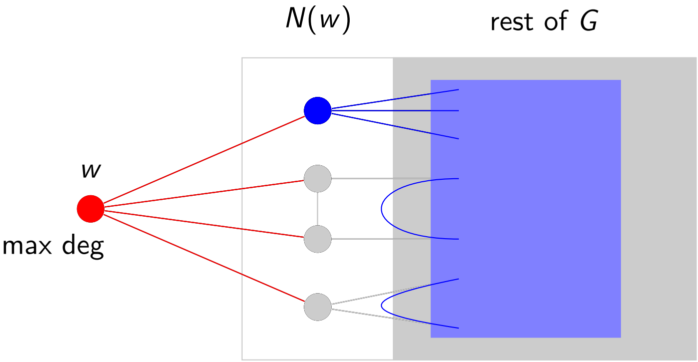

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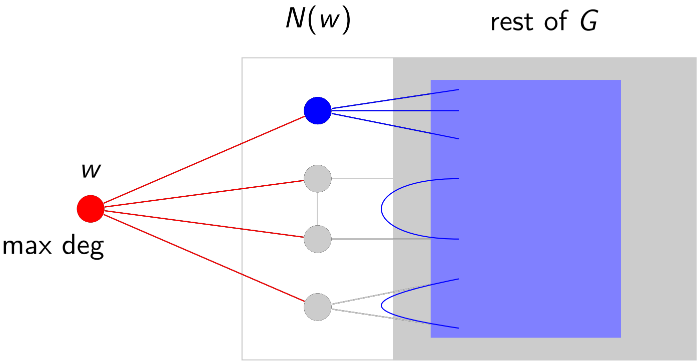

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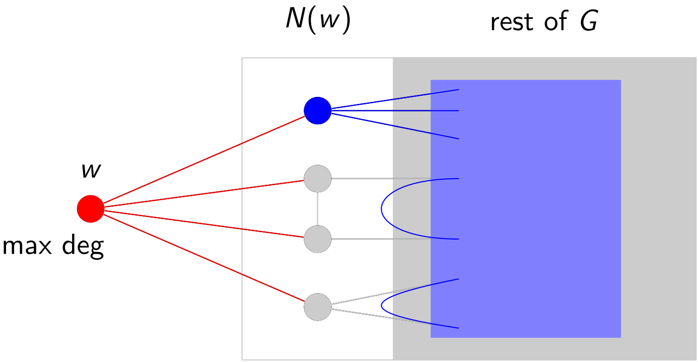

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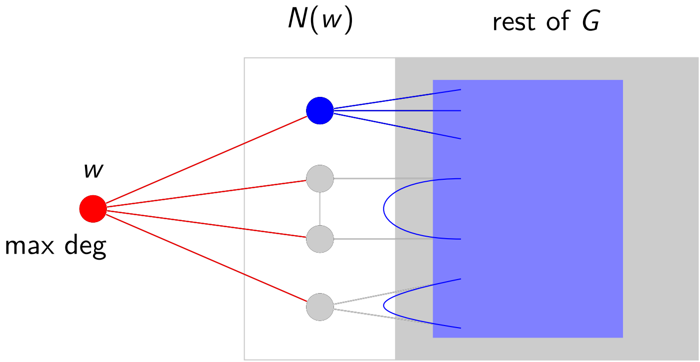

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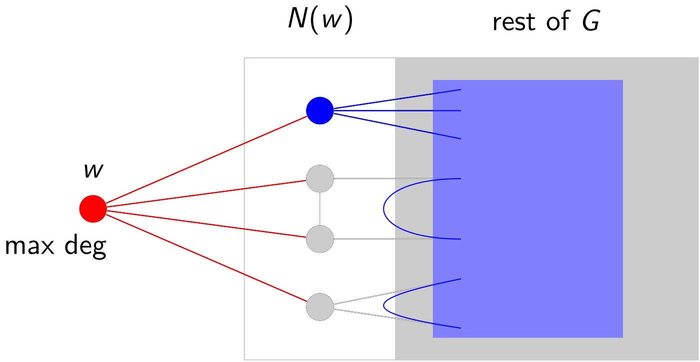

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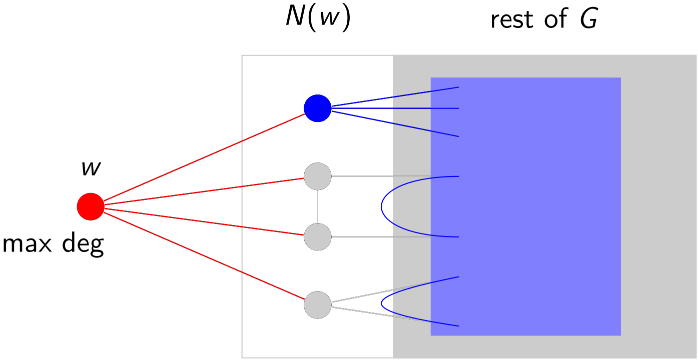

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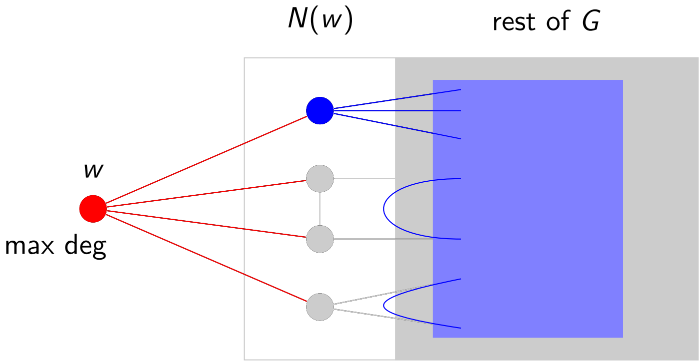

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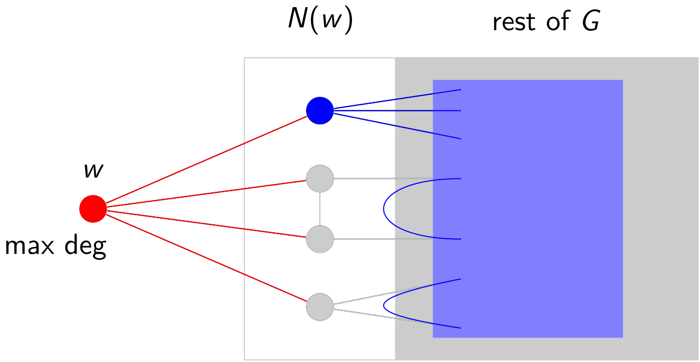

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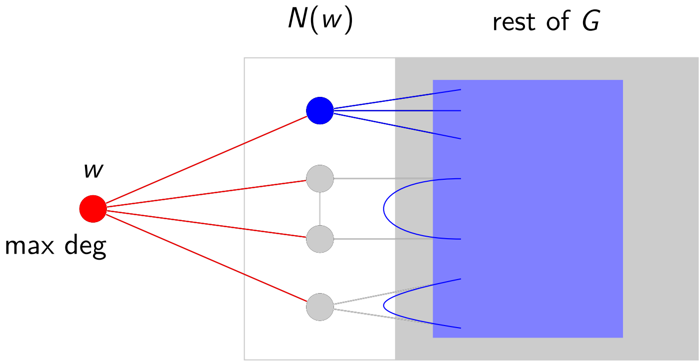

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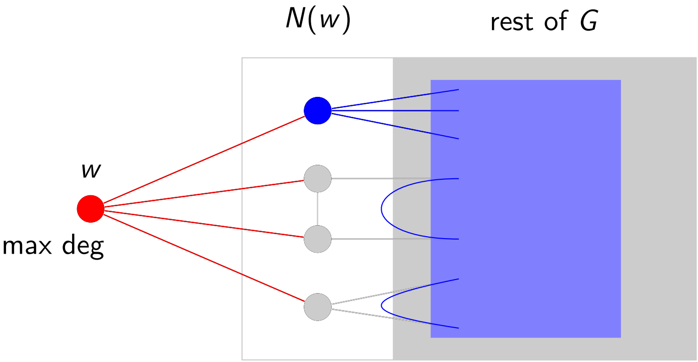

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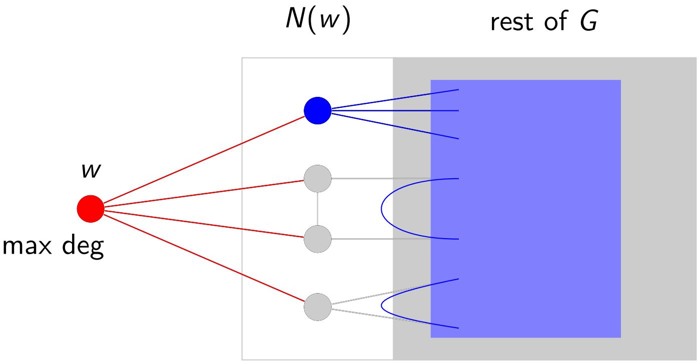

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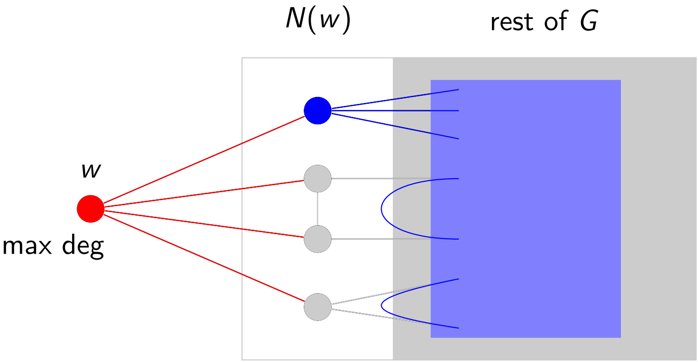

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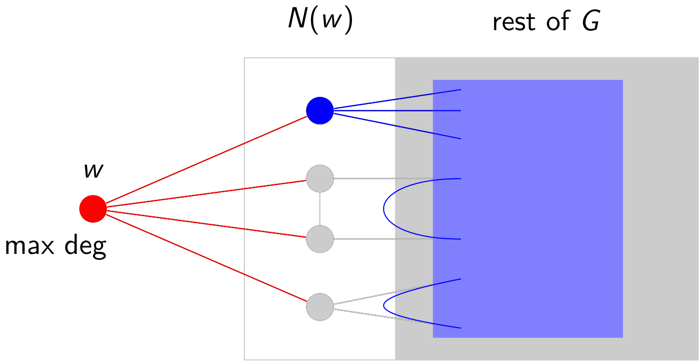

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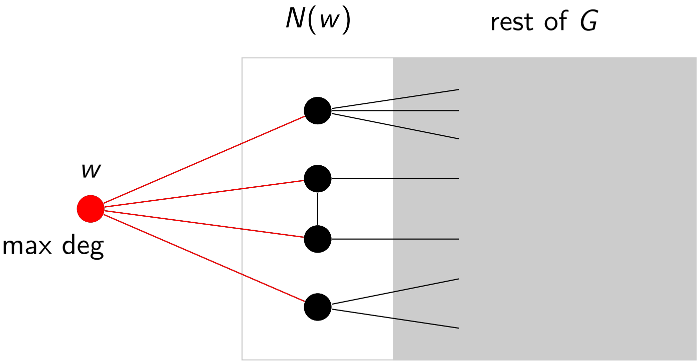

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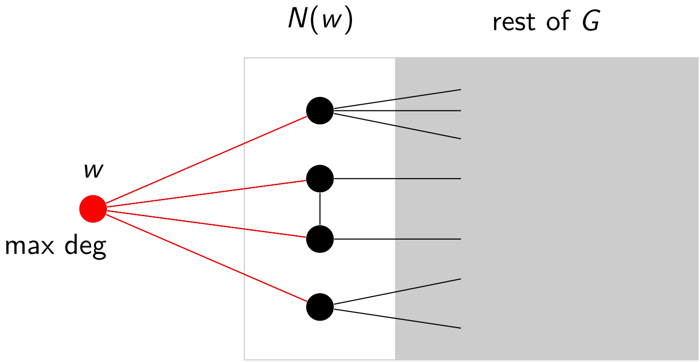
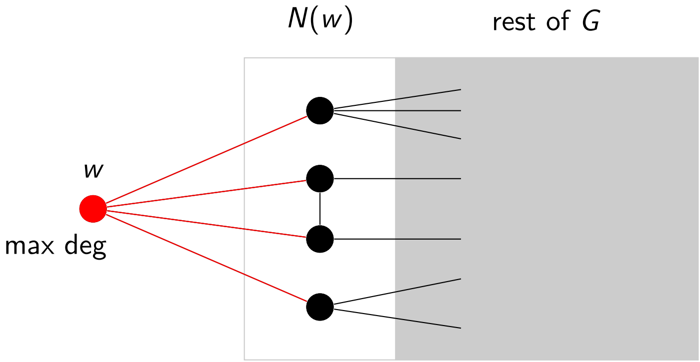


$$|R(G)| \le \underbrace{1 + \sum_{v \in V(G-w)} \frac{d_{G-w}(v) - 2}{d_{G-w}(v) + 1}}_{\displaystyle \sum_{v \in N(w)} \frac{d_{G-w}(v) - 2}{d_{G-w}(v) + 1} + \sum_{v \in V(G-w-N(w))} \frac{d_{G-w}(v) - 2}{d_{G-w}(v) + 1}}$$



$$|R(G)| \leq \quad 1 \; +$$

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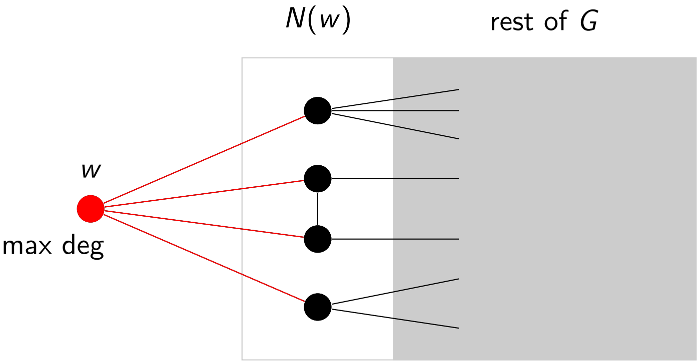

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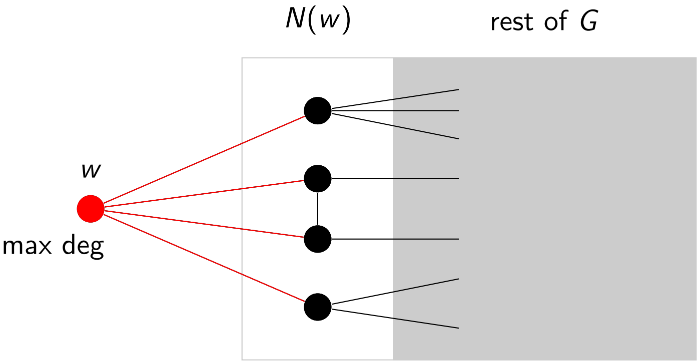

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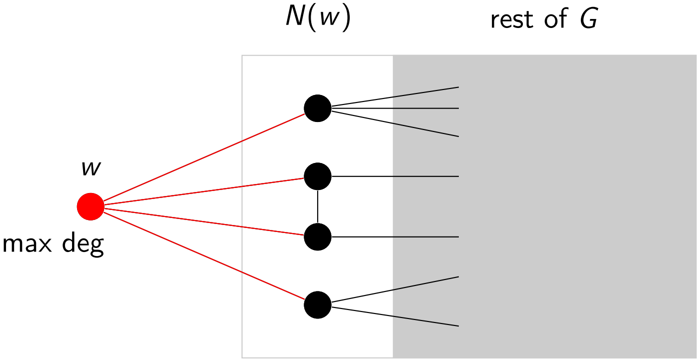

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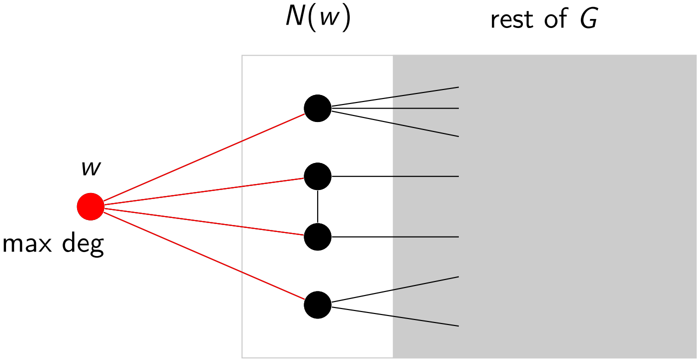
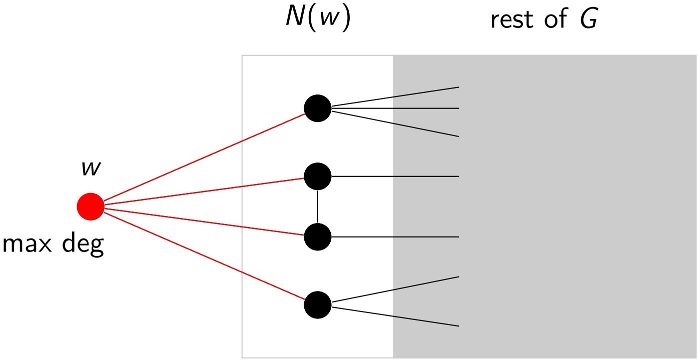


$$|R(G)| \leq 1 + \sum_{v \in N(w)} \frac{d(v)-3}{d(v)} + \sum_{v \in V(G-w-N(w))} \frac{d(v)-2}{d(v)+1}$$

$$\sum_{v \in N(w)} \frac{d(v)-1-2}{d(v)-1+1} + \sum_{v \in V(G-w-N(w))} \frac{d(v)-2}{d(v)+1}$$



$$|R(G)| \leq 1 + \sum_{v \in N(w)} \frac{d(v)-3}{d(v)} + \sum_{v \in V(G-w-N(w))} \frac{d(v)-2}{d(v)+1}$$

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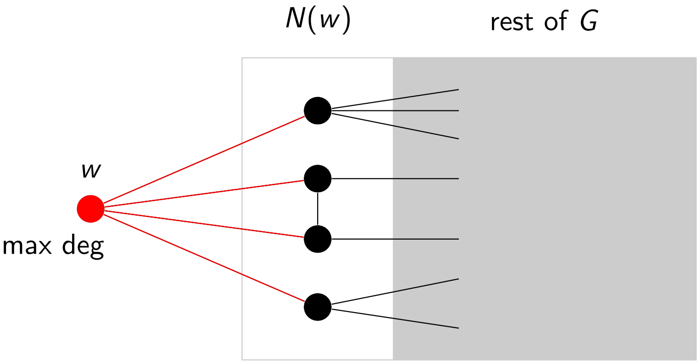
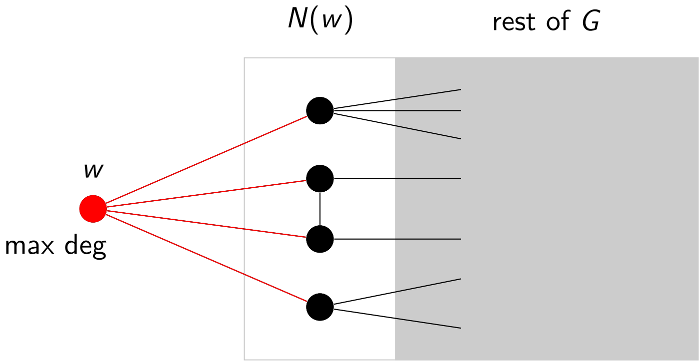


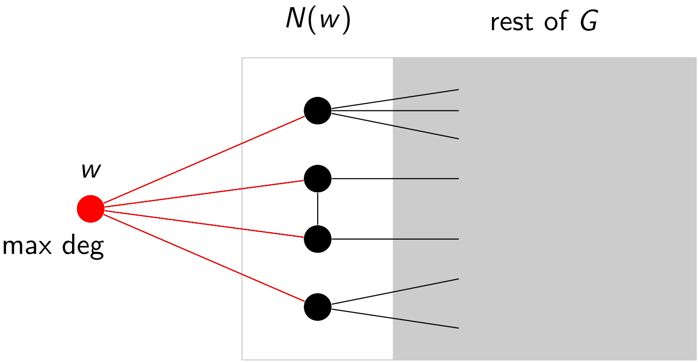
$$|R(G)| \leq 1 + \sum_{v \in N(w)} \frac{d(v) - 3}{d(v)} + \sum_{v \in V(G-w-N(w))} \frac{d(v) - 2}{d(v) + 1}$$

$$\sum_{v \in N(w)} \frac{d \quad (v) - 1 - 2}{d \quad (v) - 1 + 1} + \sum_{v \in V(G-w-N(w))} \frac{d \quad (v) - 2}{d \quad (v) + 1}$$



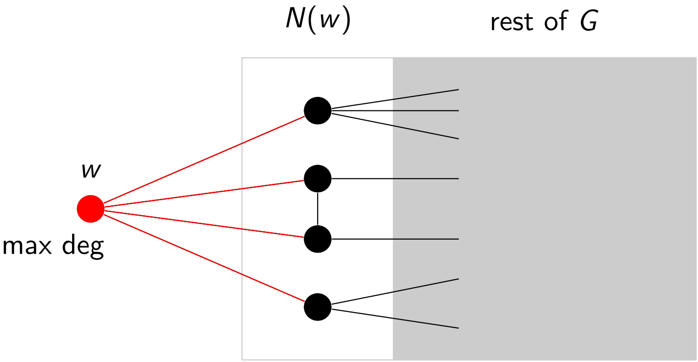
$$|R(G)| \le \quad 1 \; + \sum_{v \in N(w)} \frac{d(v) - 3}{d(v)} + \sum_{v \in V(G-w-N(w))} \frac{d(v) - 2}{d(v) + 1}$$

$$\sum_{v \in N(w)} \frac{d \qquad (v) - 1 - 2}{d \qquad (v) - 1 + 1} + \sum_{v \in V(G-w-N(w))} \frac{d \qquad (v) - 2}{d \qquad (v) + 1}$$



$$|R(G)| \leq \quad 1 \; + \sum_{v \in N(w)} \frac{d(v)-3}{d(v)} + \sum_{v \in V(G-w-N(w))} \frac{d(v)-2}{d(v)+1}$$

$$\sum_{v \in N(w)} \frac{d \qquad (v)-1-2}{d \qquad (v)-1+1} + \sum_{v \in V(G-w-N(w))} \frac{d \qquad (v)-2}{d \qquad (v)+1}$$



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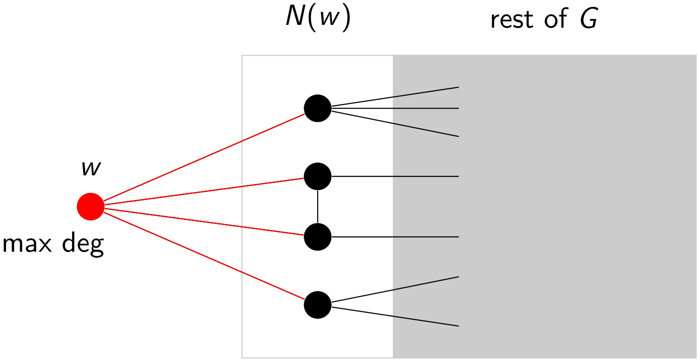
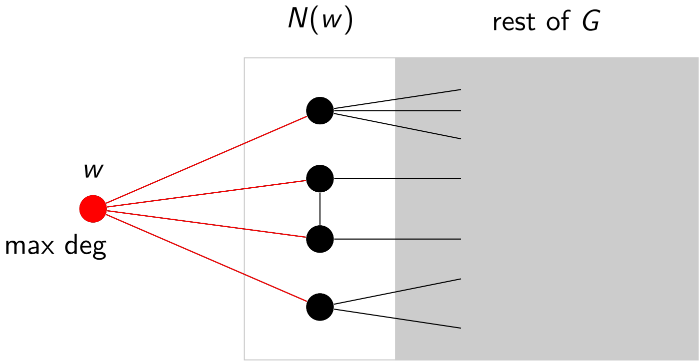


$$|R(G)| \leq 1 + \sum_{v \in N(w)} \frac{d(v)-3}{d(v)} + \sum_{v \in V(G-w-N(w))} \frac{d(v)-2}{d(v)+1}$$

$$\sum_{v \in N(w)} \frac{d\quad(v)-1-2}{d\quad(v)-1+1} + \sum_{v \in V(G-w-N(w))} \frac{d\quad(v)-2}{d\quad(v)+1}$$



$$|R(G)| \leq 1 + \sum_{v \in N(w)} \frac{d(v)-3}{d(v)} + \sum_{v \in V(G-w-N(w))} \frac{d(v)-2}{d(v)+1}$$

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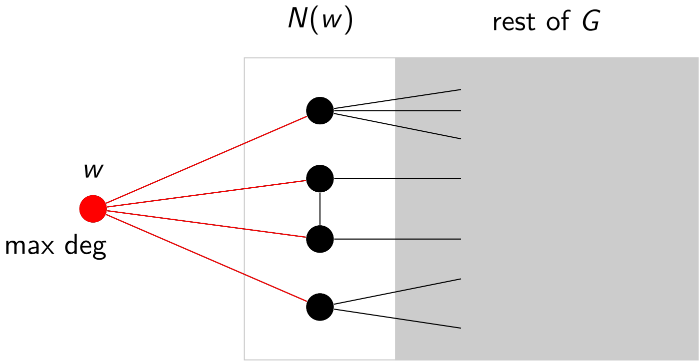
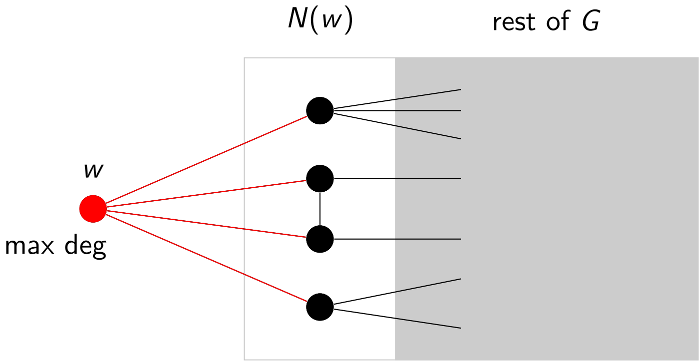


$$|R(G)| \leq \quad 1 \; + \sum_{v \in N(w)} \frac{d(v)-3}{d(v)} + \sum_{v \in V(G-w-N(w))} \frac{d(v)-2}{d(v)+1}$$



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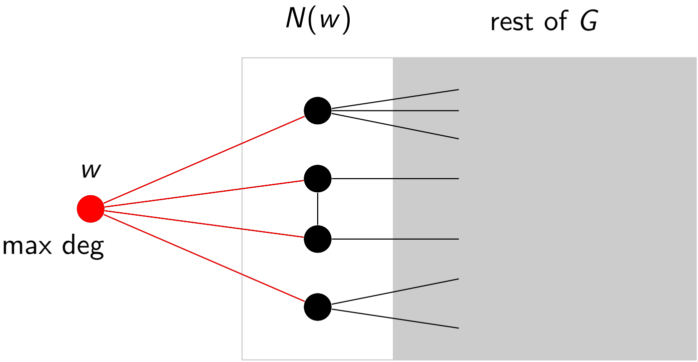
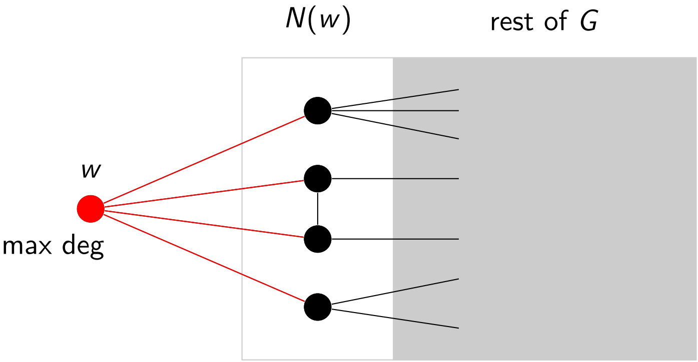


$$|R(G)| \leq 1 + \sum_{v \in N(w)} \frac{d(v)-3}{d(v)} + \sum_{v \in V(G-w-N(w))} \frac{d(v)-2}{d(v)+1}$$

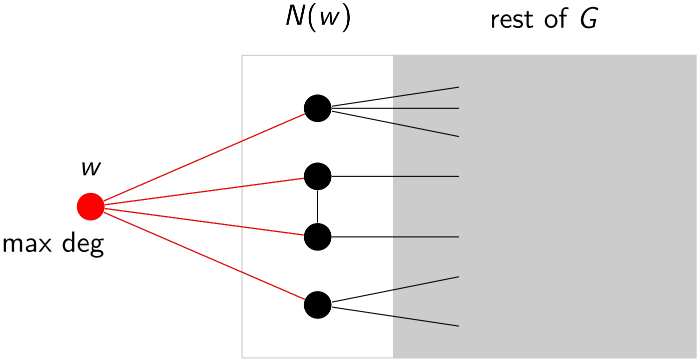
$$\underbrace{\qquad\qquad}$$

$$\sum_{v \in N(w)} \left(\frac{d(v)-3}{d(v)} - \frac{d(v)-2}{d(v)+1} \right) + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



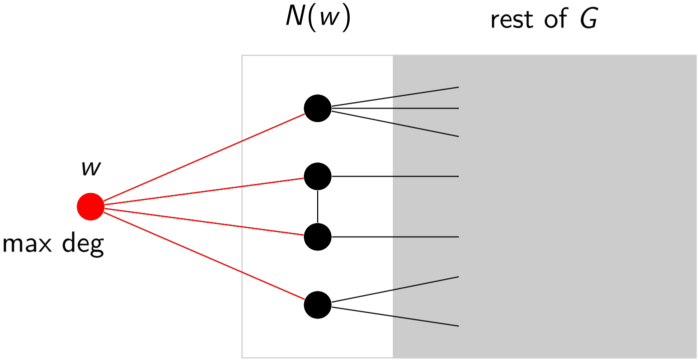
$$|R(G)| \leq \quad 1 \; +$$

$$\sum_{v \in N(w)} \left(\frac{d(v)-3}{d(v)} - \frac{d(v)-2}{d(v)+1} \right) + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



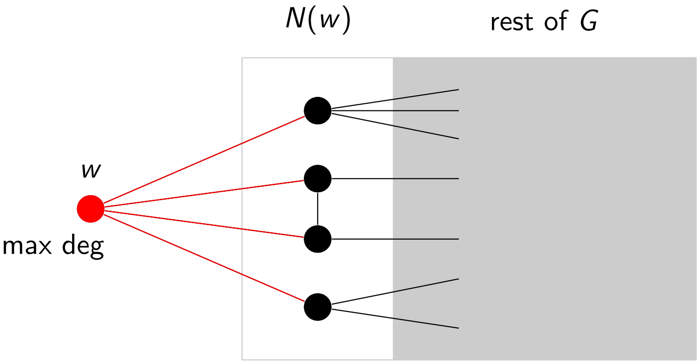
$$|R(G)| \leq \quad 1 \; + \qquad\qquad + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$

$$\sum_{v \in N(w)} \left(\frac{d(v)-3}{d(v)} - \frac{d(v)-2}{d(v)+1} \right) + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



$$|R(G)| \leq 1 - \sum_{v \in N(w)} \frac{3}{d(v)(d(v)+1)} + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$

$$\sum_{v \in N(w)} \left(\frac{d(v)-3}{d(v)} - \frac{d(v)-2}{d(v)+1} \right) + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



$$|R(G)| \leq \quad 1 - \sum_{v \in N(w)} \frac{3}{d(v)(d(v)+1)} + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$

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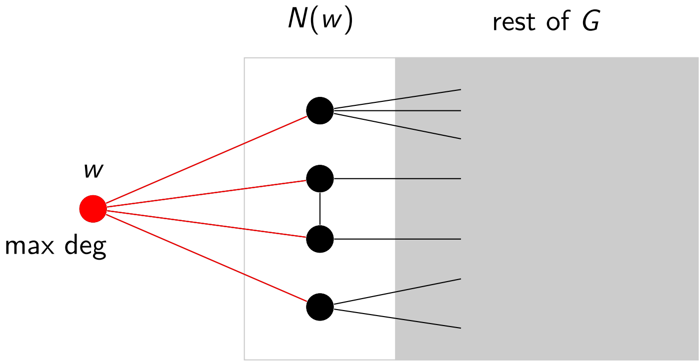
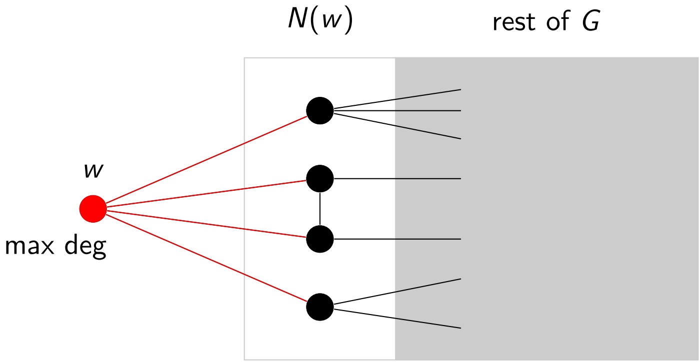


$$|R(G)| \leq \quad 1 - \sum_{v \in N(w)} \frac{3}{d(\ v\ )(d(\ v\ )+1)} + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$

$$\sum_{v \in N(w)} \left(\frac{d(v)-3}{d(v)} - \frac{d(v)-2}{d(v)+1} \right) + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



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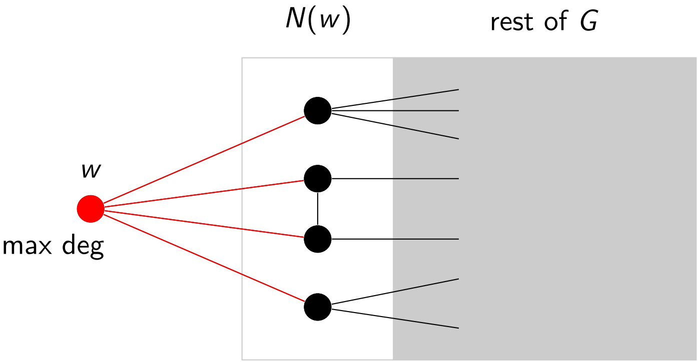

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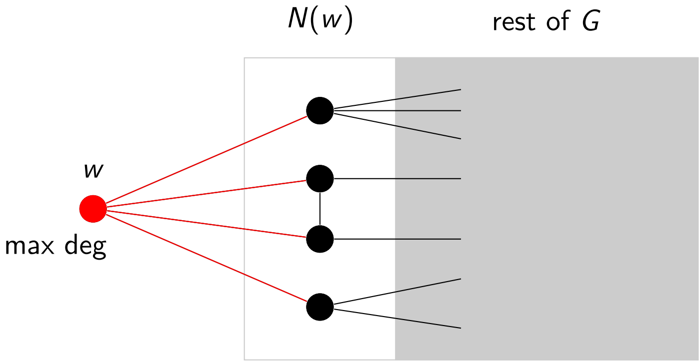

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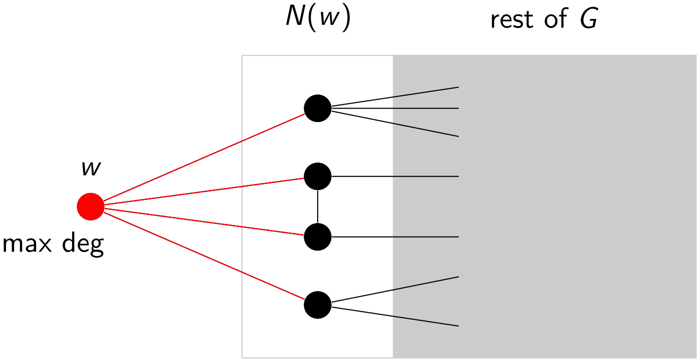

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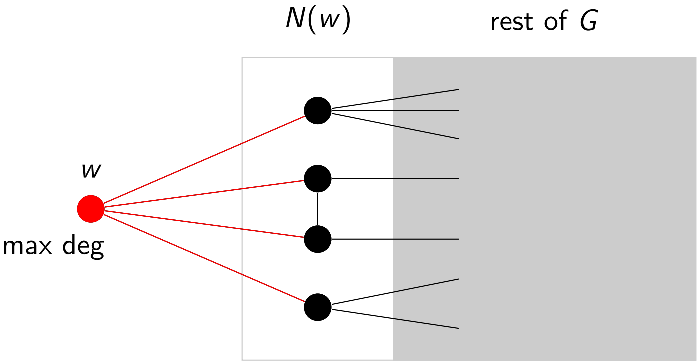

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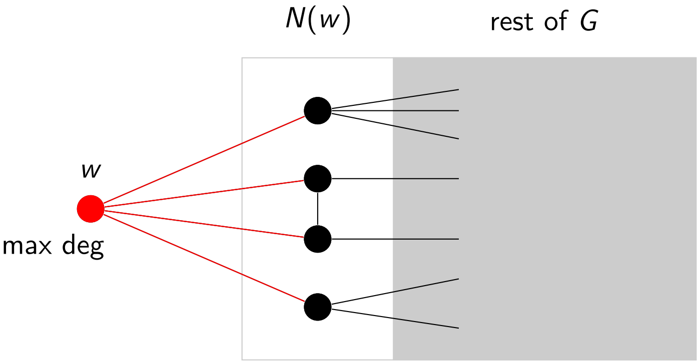
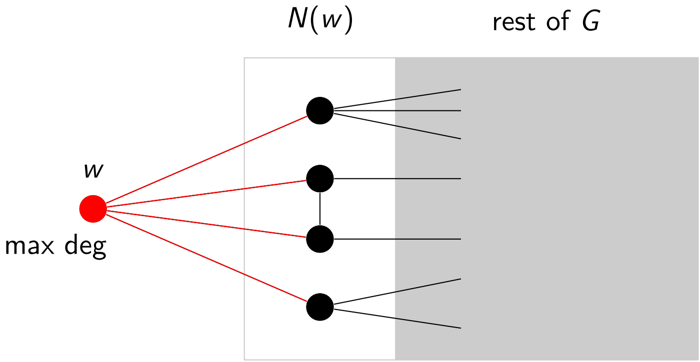
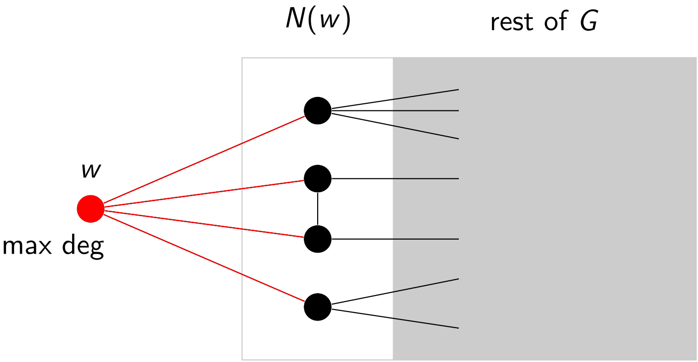


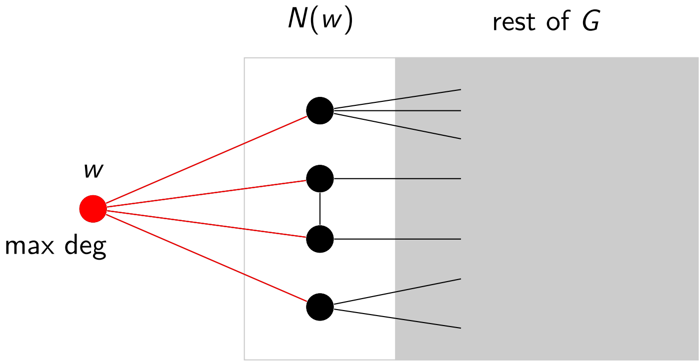
$$|R(G)| \leq 1 - \sum_{v \in N(w)} \frac{3}{d(\text{ v })(d(\text{ v }) + 1)} + \sum_{v \in V(G-w)} \frac{d(v) - 2}{d(v) + 1}$$



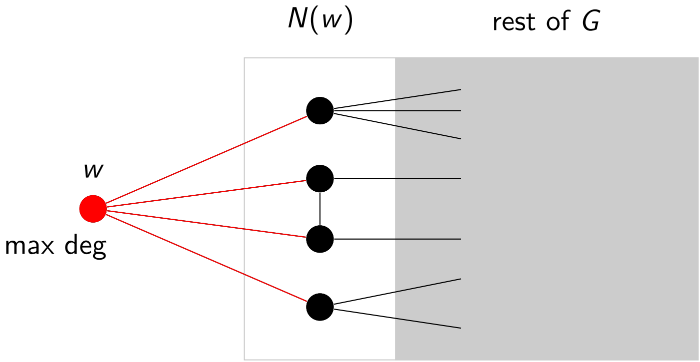
$$|R(G)| \leq \quad 1 - \sum_{v \in N(w)} \frac{3}{d(\ v\ )(d(\ v\ )+1)} + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



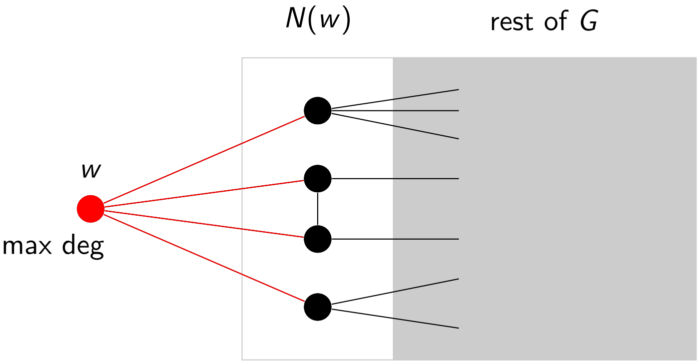
$$|R(G)| \leq \quad 1 - \sum_{v \in N(w)} \frac{3}{d(w)(d(w)+1)} + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



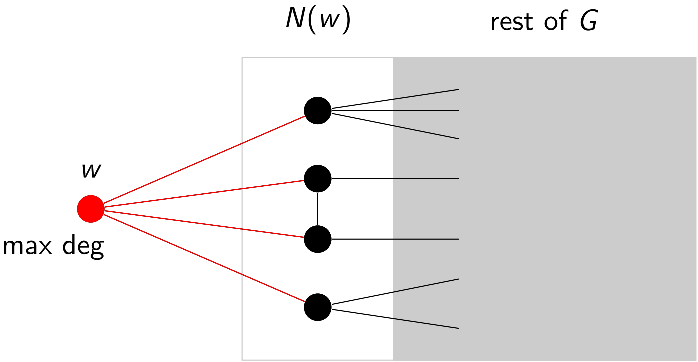
$$|R(G)| \le \quad 1 - \quad \frac{d(w) \quad 3}{d(w)(d(w)+1)} + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



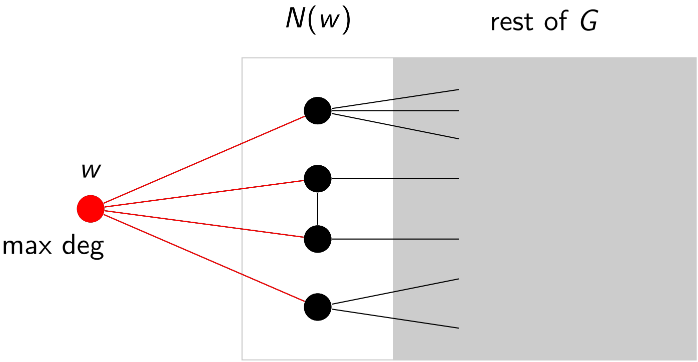
$$|R(G)| \leq 1 - \frac{3}{(d(w)+1)} + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



$$|R(G)| \leq \underbrace{1 - \frac{3}{(d(\textcolor{red}{w})+1)}}_{\frac{d(w)-2}{d(w)+1}} + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



$$|R(G)| \leq \underbrace{1 - \frac{3}{(d(w)+1)}}_{\frac{d(w)-2}{d(w)+1}} + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



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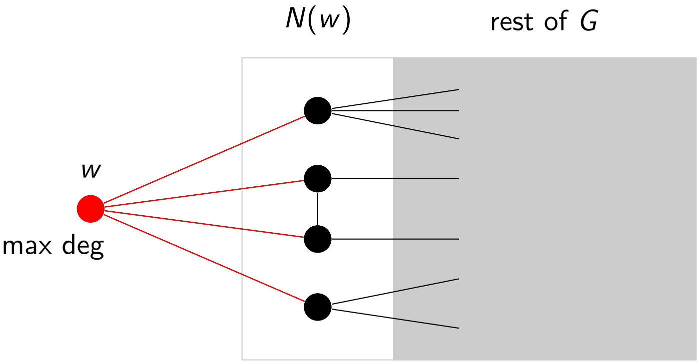

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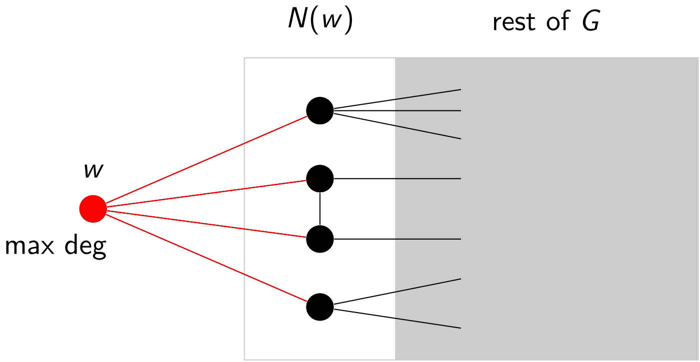

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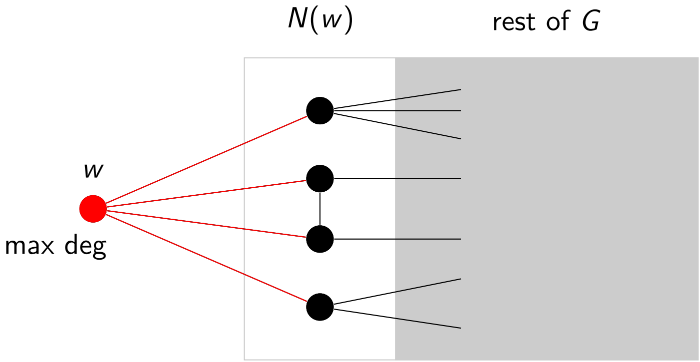

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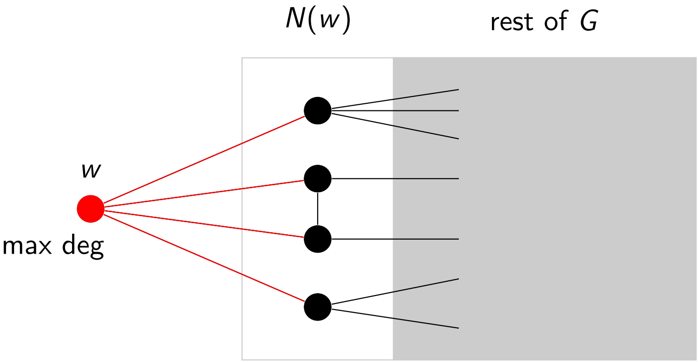
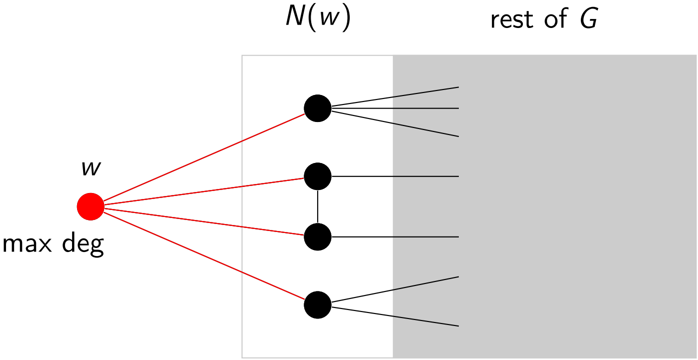


$$|R(G)| \leq \underbrace{1 - \frac{3}{(d(w)+1)}}_{\frac{d(w)-2}{d(w)+1}} + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



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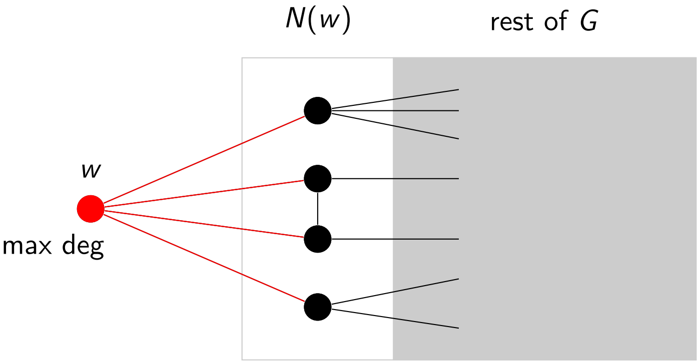

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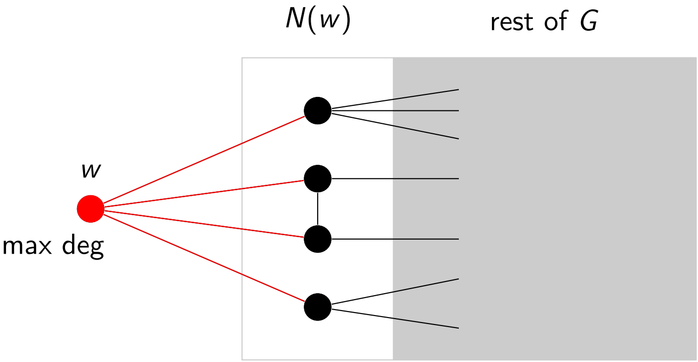
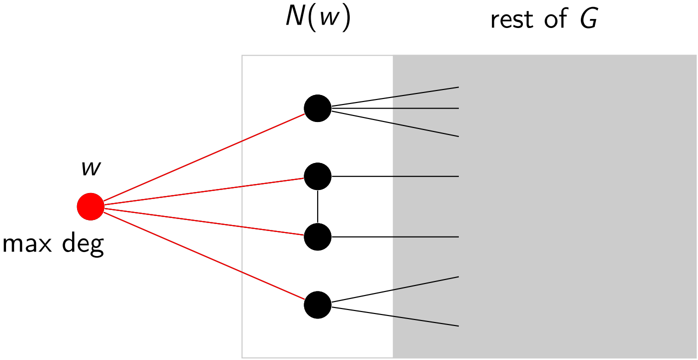
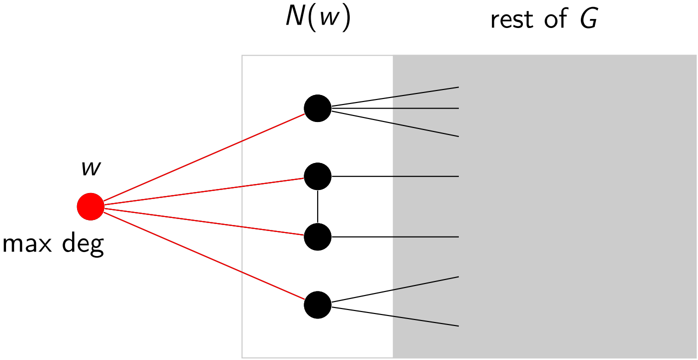


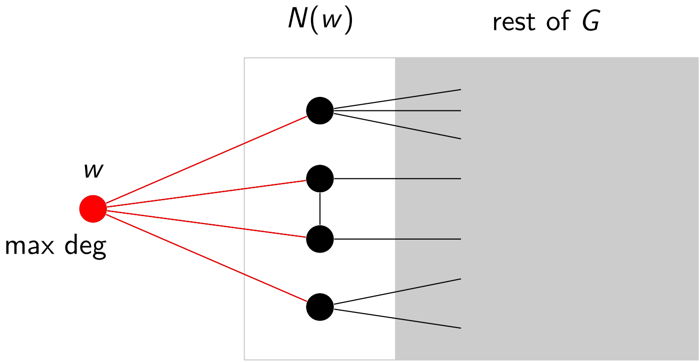
$$|R(G)| \le \underbrace{1 - \frac{3}{(d(w)+1)}}_{\frac{d(w)-2}{d(w)+1}} + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



$$|R(G)| \leq \underbrace{1 - \frac{3}{(d(w)+1)}}_{\dfrac{d(w)-2}{d(w)+1}} + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$

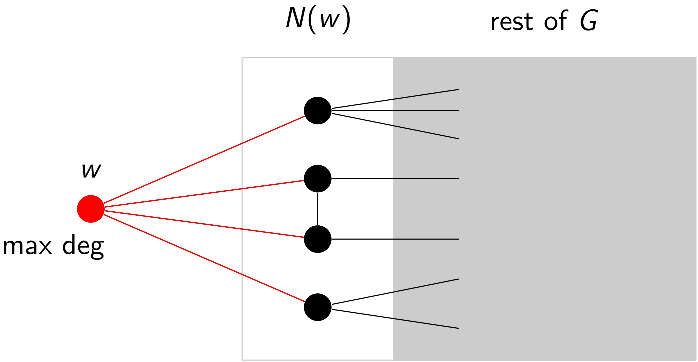


$$|R(G)| \leq \underbrace{1 - \frac{3}{(d(w)+1)}}_{\frac{d(w)-2}{d(w)+1}} + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$



$$|R(G)| \leq \qquad + \sum_{v \in V(G-w)} \frac{d(v)-2}{d(v)+1}$$

$$\frac{d(w)-2}{d(w)+1}$$



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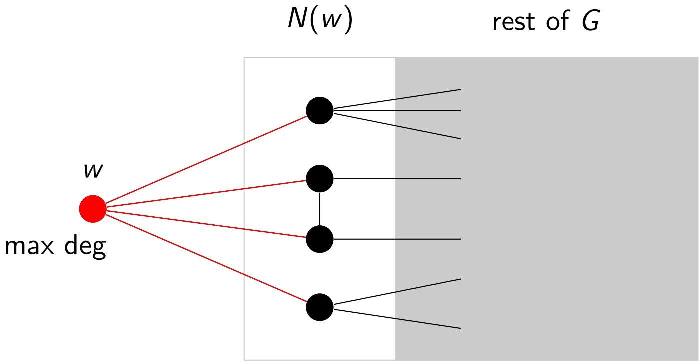

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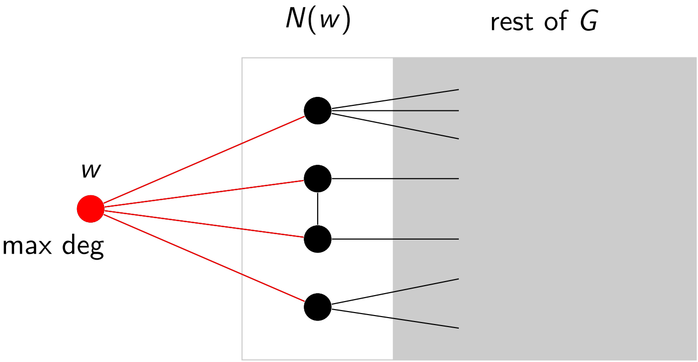

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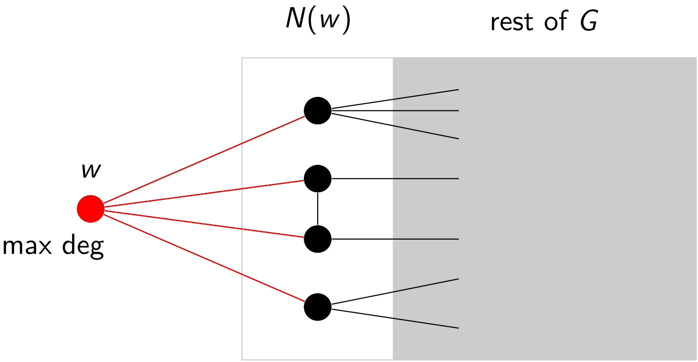

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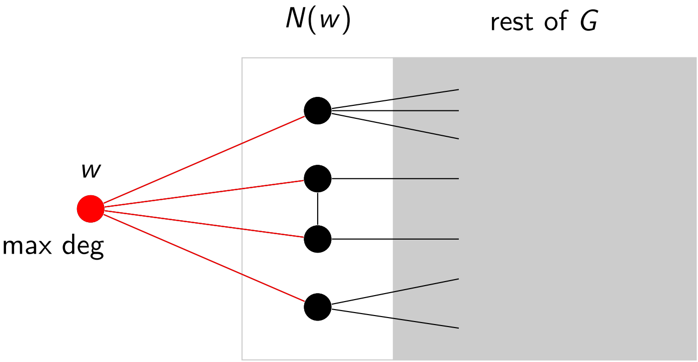

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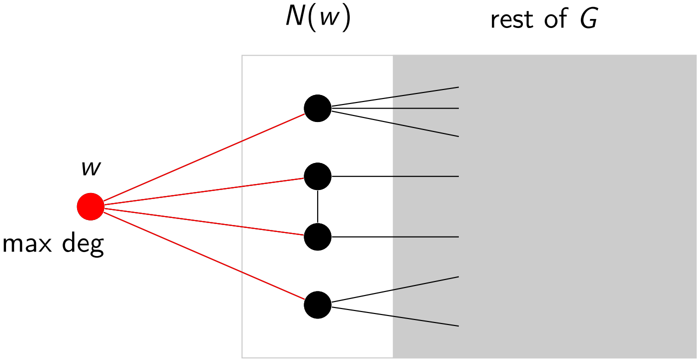

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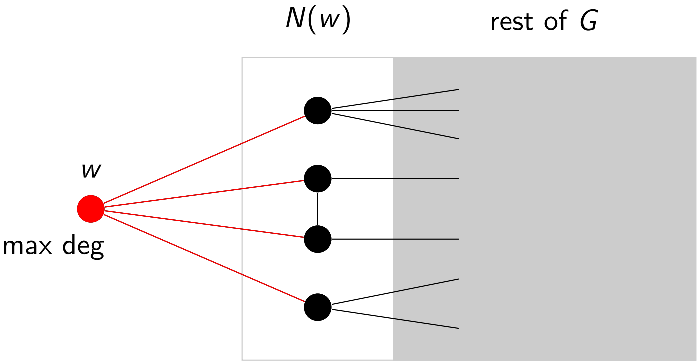

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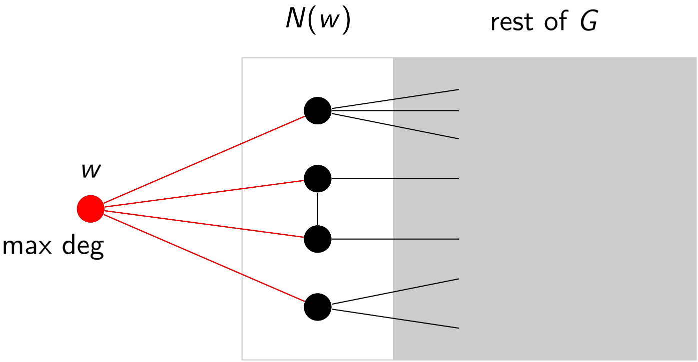

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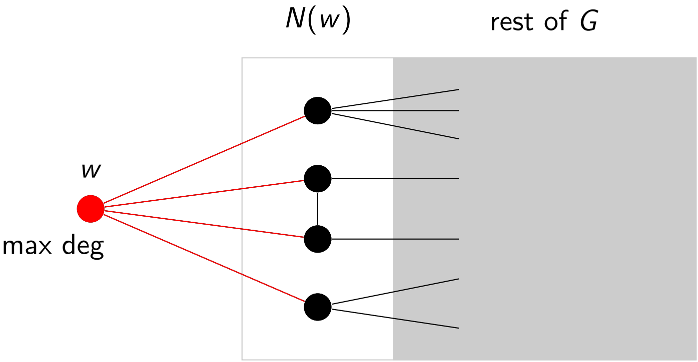

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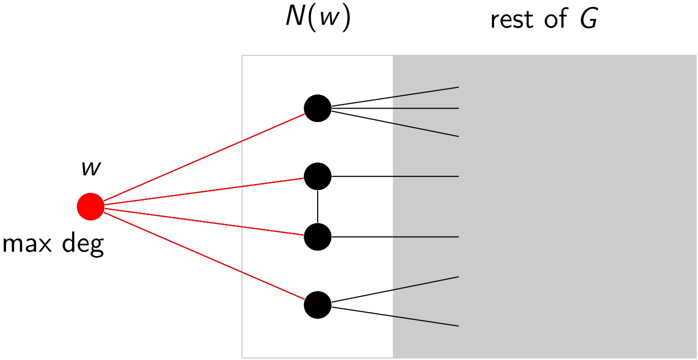

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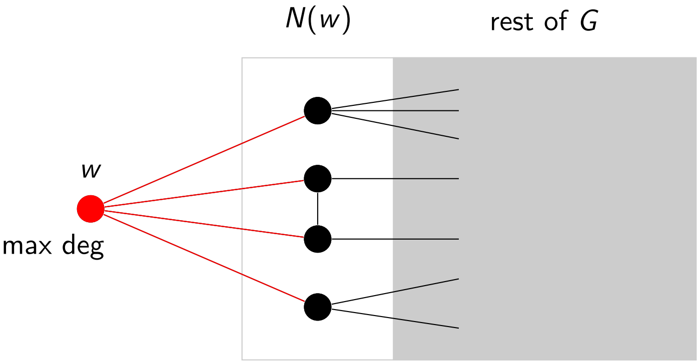

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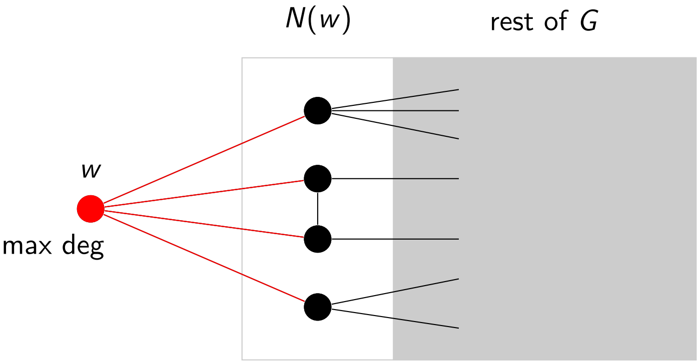

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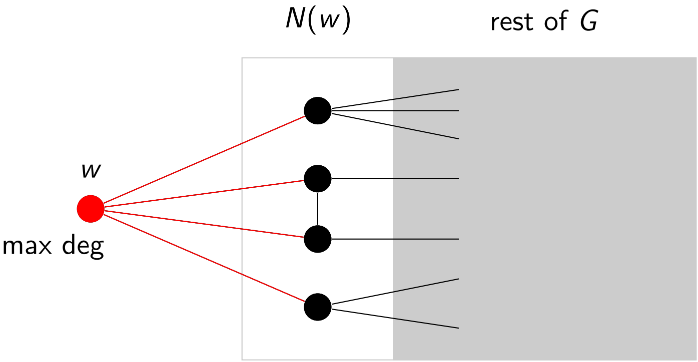

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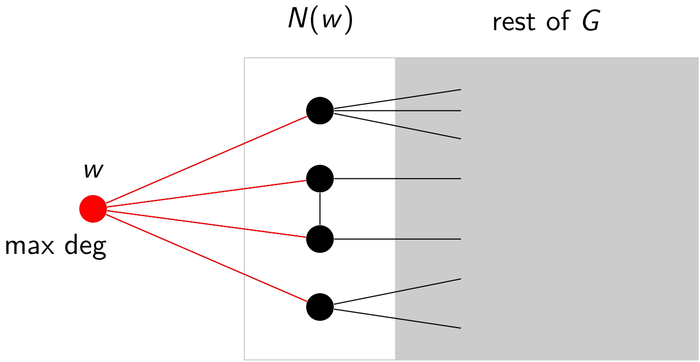

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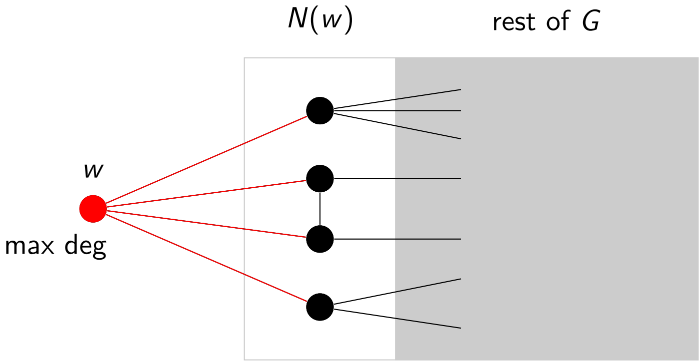

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Can use this result to analyse behaviour of Algorithm 1 for graphs of given average degree:

Theorem (E & F, 2003, 2007)

If G is has average degree $d \geq 4$, or is connected and has average degree $d \geq 2$, then Algorithm 1 finds and induced planar subgraph of G of at least

$$\left(\frac{3}{d+1} + \frac{3(d - \lfloor d \rfloor)(\lceil d \rceil - d)}{(d+1)(\lfloor d \rfloor + 1)(\lceil d \rceil + 1)}\right) n$$

vertices.

Time complexity $= O(nm)$.

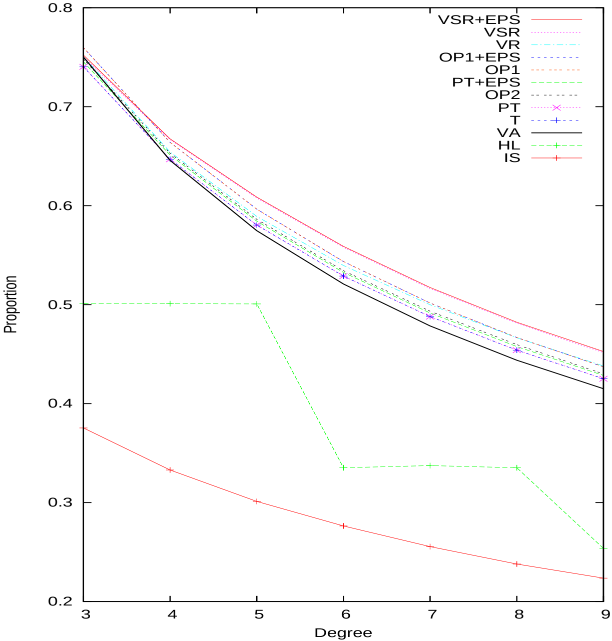
Experiments

- Algorithms: Independent Set (IS), Induced Forest (T), Halldórsson-Lau (HL), Vertex Addition (VA), Outerplanar (OP2), Vertex Removal (VR), ...
- $n = 20, \ldots, 10000$
- $d = 3, 4, \ldots, 9$
- random graphs:
  d-regular (Steger-Wormald),
  expected average degree d (classical)
- number of graphs of each type:
  50 (for $n \leq 1000$), 20 (for $n \geq 1000$)

Further information:

- Morgan & Farr, *JGAA*, to appear (2007)
- `http://www.csse.monash.edu.au/~kmorgan/MIPS.html`

Performance versus degree: random d-regular graphs



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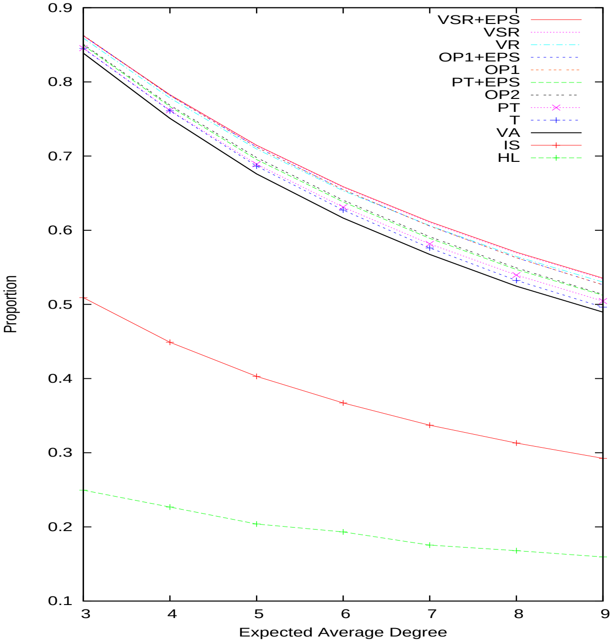
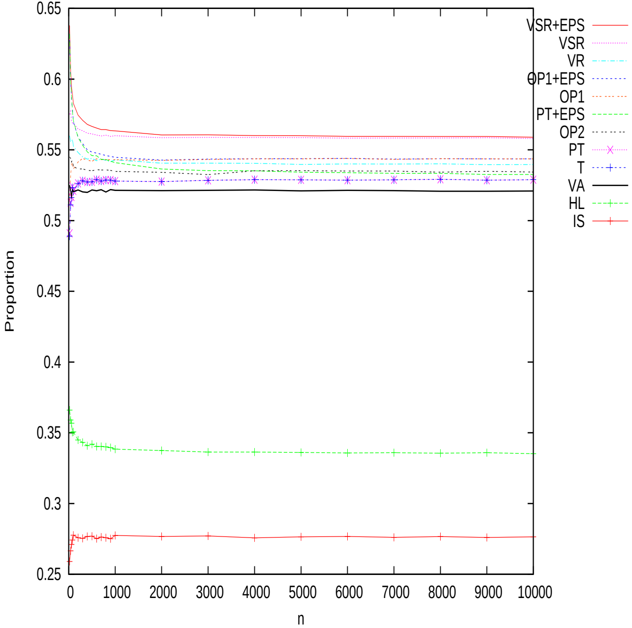


Performance versus n: random d-regular graphs



MIPS and fragmentability

MIPS is useful for breaking graphs into small pieces.
Given G, with max/ave degree $\leq d$:

1. remove vertices from G to leave induced planar subgraph $\langle P \rangle$;
2. remove $o(n)$ vertices from $\langle P \rangle$ to leave bounded size pieces (e.g., apply Planar Separator Theorem (Lipton & Tarjan) recursively).

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Converely, bounds on fragmentability can give *upper* bound on size of MIPS.
E.g., for $d = 3$, cannot do better than $\dfrac{3}{d+1} = \dfrac{3}{4}$.

For more info on fragmentability:
Edwards & Farr (2001, 2007),
Haxell, Pikhurko & Thomason (preprint)

Future work

- Improve lower bound on proportion of vertices in MIPS.

Our best: $\dfrac{3}{d+1}$.

Ceiling: $\dfrac{4}{d+1}$. Consider $K_4 \leq K_{d+1}$.

How close to ceiling can we get? Is there a lower ceiling?

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- Experimental comparison with maxim*al* induced planar subgraph.