$\ell\text{-link graphs}$

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A study of ℓ -link graphs

PhD Candidate: Bin Jia Supervisor: David R. Wood

Discrete Maths Research Group Seminar Monash University, 30 May 2014

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Graphs

All graphs are loopless. Parallel edges are considered to be different.

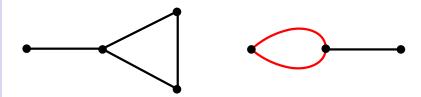


Figure: A simple graph and a graph with parallel edges

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What is an ℓ-link?

An ℓ -link is a walk of length $\ell \geqslant 0$ such that consecutive edges are different.

• 0-link • 1-link



Figure: vertices, edges, paths are all ℓ-links

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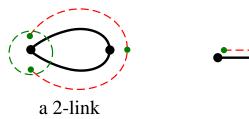
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Some complicated ℓ -links

An ℓ -link is a walk of length $\ell \geqslant 0$ such that consecutive edges are different.



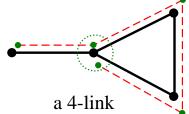


Figure: ℓ-links with repeated vertices

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More complicated ℓ-links

An ℓ -link is a walk of length $\ell \geqslant 0$ such that consecutive edges are different.

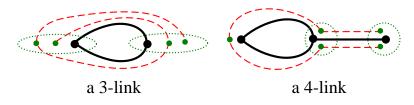


Figure: ℓ-links with repeated edges

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Counterexamples

An ℓ -link is a walk of length $\ell \geqslant 0$ such that consecutive edges are different.

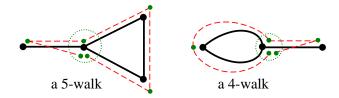


Figure: Walks that are not ℓ-links

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Shunting *ℓ*-links

An ℓ -link can be shunted to another ℓ -link in one step through an $(\ell+1)$ -link.



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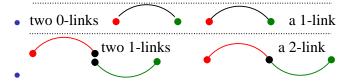
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Shunting ℓ -links

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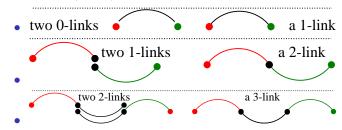
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Shunting ℓ -links

An ℓ -link can be shunted to another ℓ -link in one step through an $(\ell + 1)$ -link.



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ℓ-link simple graphs

The ℓ -link simple graph of a graph G is the graph with

vertices the ℓ-links of G; two ℓ-links are adjacent if

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ℓ -link simple graphs

- vertices the ℓ-links of G; two ℓ-links are adjacent if
 - one can be shunted onto the other through an $(\ell+1)$ -link of G.

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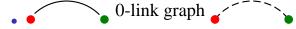
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ℓ-link simple graphs

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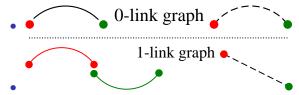
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ℓ-link simple graphs

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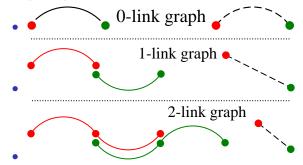
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ℓ -link simple graphs

- vertices the ℓ-links of G; two ℓ-links are adjacent if
- one can be shunted onto the other through an $(\ell+1)$ -link of G.



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ℓ-link graphs

• Two ℓ -links L and R of a graph G might be shunted to each other through $\mu_G(L,R)$ different $(\ell+1)$ -links.

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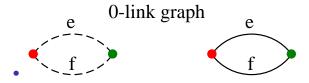
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ℓ-link graphs

• Two ℓ -links L and R of a graph G might be shunted to each other through $\mu_G(L,R)$ different $(\ell+1)$ -links.



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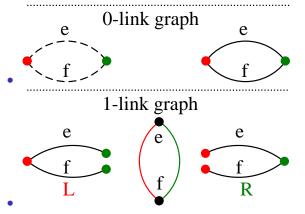
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• Two ℓ -links L and R of a graph G might be shunted to each other through $\mu_G(L,R)$ different $(\ell+1)$ -links.



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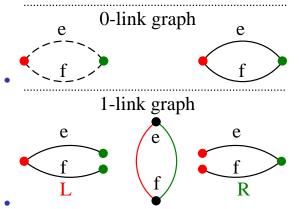
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ℓ-link graphs

• Two ℓ -links L and R of a graph G might be shunted to each other through $\mu_G(L,R)$ different $(\ell+1)$ -links.



For any two ℓ-links L and R of G, there are μ_G(L, R) parallel edges between them in the ℓ-link graph L_ℓ(G) of G.

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Path graphs

In the definition of ℓ -link simple graph, if the vertices are ℓ -paths of G, then the constructed graph is the ℓ -path graph $\mathbb{P}_{\ell}(G)$ introduced by Broersma and Hoede in 1989.

• By definition $\mathbb{L}_0(G)=G$ and $\mathbb{P}_1(G)=\mathbb{L}(G)$ (Broersma and Hoede, 1989).

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- By definition $\mathbb{L}_0(G) = G$ and $\mathbb{P}_1(G) = \mathbb{L}(G)$ (Broersma and Hoede, 1989).
- If $\ell \in \{0,1\}$, then $\mathbb{P}_{\ell}(G)$ is isomorphic to the underlying simple graph of $\mathbb{L}_{\ell}(G)$.

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- By definition $\mathbb{L}_0(G) = G$ and $\mathbb{P}_1(G) = \mathbb{L}(G)$ (Broersma and Hoede, 1989).
- If $\ell \in \{0,1\}$, then $\mathbb{P}_{\ell}(G)$ is isomorphic to the underlying simple graph of $\mathbb{L}_{\ell}(G)$.
- If $\ell \geqslant 2$, then $\mathbb{P}_{\ell}(G)$ is an induced subgraph of $\mathbb{L}_{\ell}(G)$.

Path graphs

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- By definition $\mathbb{L}_0(G) = G$ and $\mathbb{P}_1(G) = \mathbb{L}(G)$ (Broersma and Hoede, 1989).
- If $\ell \in \{0,1\}$, then $\mathbb{P}_{\ell}(G)$ is isomorphic to the underlying simple graph of $\mathbb{L}_{\ell}(G)$.
- If $\ell \geqslant 2$, then $\mathbb{P}_{\ell}(G)$ is an induced subgraph of $\mathbb{L}_{\ell}(G)$.
- If G is simple and $\ell \in \{0, 1, 2\}$, then $\mathbb{P}_{\ell}(G) = \mathbb{L}_{\ell}(G)$.

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- By definition $\mathbb{L}_0(G) = G$ and $\mathbb{P}_1(G) = \mathbb{L}(G)$ (Broersma and Hoede, 1989).
- If $\ell \in \{0,1\}$, then $\mathbb{P}_{\ell}(G)$ is isomorphic to the underlying simple graph of $\mathbb{L}_{\ell}(G)$.
- If $\ell \geqslant 2$, then $\mathbb{P}_{\ell}(G)$ is an induced subgraph of $\mathbb{L}_{\ell}(G)$.
- If G is simple and $\ell \in \{0, 1, 2\}$, then $\mathbb{P}_{\ell}(G) = \mathbb{L}_{\ell}(G)$.
- If $girth(G) > \ell \geqslant 2$ then $\mathbb{P}_{\ell}(G) = \mathbb{L}_{\ell}(G)$.

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ℓ-arc graphs

Godsil and Royle [Algebraic graph theory] defined the ℓ -arc graph $\mathbb{A}_{\ell}(G)$ of G as the digraph with vertices the ℓ -arcs of G. If an ℓ -arc \vec{L} can be shunted to another \vec{R} in one step, then there is an arc from \vec{L} to \vec{R} in $\mathbb{A}_{\ell}(G)$.

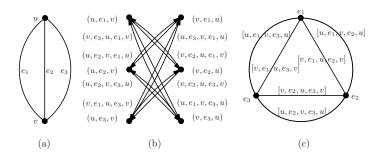


Figure: A multigraph, its 1-arc graph, and 1-link graph

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Incidence patterns

Introduced by Grünbaum (1969), an incidence pattern is a function that maps given graphs or similar objects to graphs. So the constructions of line graphs, ℓ -path graphs, ℓ -arc graphs and ℓ -link graphs are all incidence patterns. Two general problems have been proposed by Grunbaum as the characterization of constructed graphs and the determination of original graphs.

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R&D problems

Grünbaum's general problems for incidence pattern can be stated more precisely for ℓ -link graphs. For each integer $\ell \geqslant 0$ and every finite graph H:

Recognition problem Decide whether H is an ℓ -link graph. **Determination problem** Find the set of ℓ -roots of H.

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0-link graph

• Each simple graph is a 0-path graph of itself.

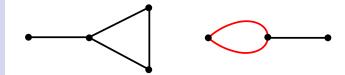


Figure: It is worthy to define the ℓ -link graphs to be multigraphs

 ℓ -link graphs

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0-link graph

- Each simple graph is a 0-path graph of itself.
- The 0-path graph of a graph G is the underlying simple graph of G.

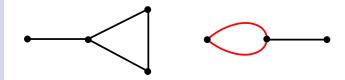


Figure: It is worthy to define the ℓ -link graphs to be multigraphs

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0-link graph

- Each simple graph is a 0-path graph of itself.
- The 0-path graph of a graph *G* is the underlying simple graph of *G*.
- Each graph is a 0-link graph of itself.

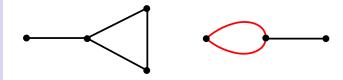


Figure: It is worthy to define the ℓ -link graphs to be multigraphs

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Line graph

[J. Krausz, 1943] A graph is a line graph of some simple graph if and only if it is simple and admits a partition of edges in which each part induces a complete subgraph so that every vertex lies in at most two of these subgraphs.

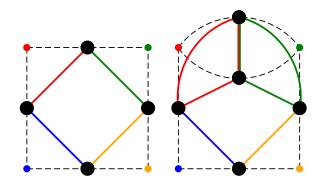


Figure: Characterization of line graphs

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Line graph

[J. Krausz, 1943] A graph is a line graph of some simple graph if and only if it is simple and admits a partition of edges in which each part induces a complete subgraph so that every vertex lies in at most two of these subgraphs.

The statement may be false for multigraphs.

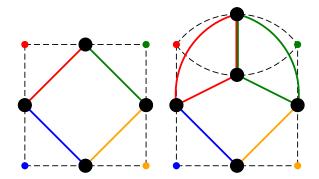


Figure: Characterization of line graphs

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1-link graph

A graph H is a 1-link graph if and only if

 E(H) can be partitioned such that each part induces a complete simple subgraph of H;

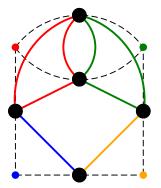


Figure: A line graph is a underlying simple graph of a 1-link graph

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1-link graph

A graph H is a 1-link graph if and only if

- E(H) can be partitioned such that each part induces a complete simple subgraph of H;
- and each vertex of *H* is in at most two such subgraphs.

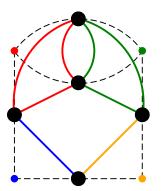


Figure: A line graph is a underlying simple graph of a 1-link graph

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line graphs VS 1-link graphs

Let *H* be a simple graph. The following are equivalent:

• *H* is a line graph (introduced by Whitney, 1932).

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line graphs VS 1-link graphs

Let *H* be a simple graph. The following are equivalent:

- *H* is a line graph (introduced by Whitney, 1932).
- H is a 1-path graph (Defined by Broersma and Hoede, 1989).

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line graphs VS 1-link graphs

Let *H* be a simple graph. The following are equivalent:

- *H* is a line graph (introduced by Whitney, 1932).
- H is a 1-path graph (Defined by Broersma and Hoede, 1989).
- By duplicating some edges of H we can obtain a 1-link graph.

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line graphs VS 1-link graphs

Let *H* be a simple graph. The following are equivalent:

- *H* is a line graph (introduced by Whitney, 1932).
- H is a 1-path graph (Defined by Broersma and Hoede, 1989).
- By duplicating some edges of H we can obtain a 1-link graph.
- By duplicating some edges we can obtain a graph H'
 which admits an edge partition such that each part
 induces a complete simple subgraph of H', and that
 each vertex of H' is in at most two such subgraphs.

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Whitney's isomorphism theorem

The line graphs of K_3 and $K_{1,3}$ are isomorphic to K_3 . The line graphs of K_0 and K_1 are isomorphic to K_0 . These are the only pairs of nonisomorphic connected graphs with isomorphic line graphs.

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Whitney's isomorphism theorem

The line graphs of K_3 and $K_{1,3}$ are isomorphic to K_3 . The line graphs of K_0 and K_1 are isomorphic to K_0 . These are the only pairs of nonisomorphic connected graphs with isomorphic line graphs.

 We characterised all nonisomorphic pairs with isomorphic 1-link graphs.

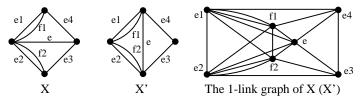


Figure: A pair of connected graphs with isomorphic 1-link graphs

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From multigraphs to simple graphs

For $s\geqslant 1$ and each graph G, $(\mathbb{L}_{\ell}(G))^{\langle s\rangle}\cong \mathbb{L}_{\ell s}(G^{\langle s\rangle})$.

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From multigraphs to simple graphs

For $s\geqslant 1$ and each graph G, $(\mathbb{L}_{\ell}(G))^{\langle s\rangle}\cong \mathbb{L}_{\ell s}(G^{\langle s\rangle})$.

• $(\mathbb{L}_1(G))^{\langle s \rangle} \cong \mathbb{L}_s(G^{\langle s \rangle}).$

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From multigraphs to simple graphs

For $s\geqslant 1$ and each graph G, $(\mathbb{L}_{\ell}(G))^{\langle s\rangle}\cong \mathbb{L}_{\ell s}(G^{\langle s\rangle})$.

- $(\mathbb{L}_1(G))^{\langle s \rangle} \cong \mathbb{L}_s(G^{\langle s \rangle}).$
- This projects multigraphs to simple graphs.

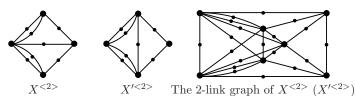


Figure: A pair of connected graphs with isomorphic 2-link (2-path) graphs

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Applications of Whitney's theorem

Xueliang Li and Yan Liu proved that there exists no triple of nonnull connected simple graphs with isomorphic connected 2-path graphs.

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Applications of Whitney's theorem

Xueliang Li and Yan Liu proved that there exists no triple of nonnull connected simple graphs with isomorphic connected 2-path graphs.

 $\bullet \ (\mathbb{L}_1(G))^{\langle 2 \rangle} \cong \mathbb{L}_2(G^{\langle 2 \rangle}).$

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Applications of Whitney's theorem

Xueliang Li and Yan Liu proved that there exists no triple of nonnull connected simple graphs with isomorphic connected 2-path graphs.

- $(\mathbb{L}_1(G))^{\langle 2 \rangle} \cong \mathbb{L}_2(G^{\langle 2 \rangle}).$
- There exists no triple of connected graphs with isomorphic 1-link graphs.

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Uniqueness of original graphs

Xueliang Li proved that simple graphs of minimum degree ≥ 3 are isomorphic if and only if their 2-path graphs are isomorphic. Li also conjectured that the assertion is true when minimum degree is 2, which was shown to be false by Aldred et al.

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Uniqueness of original graphs

Xueliang Li proved that simple graphs of minimum degree \geqslant 3 are isomorphic if and only if their 2-path graphs are isomorphic. Li also conjectured that the assertion is true when minimum degree is 2, which was shown to be false by Aldred et al.

• $(\mathbb{L}_1(G))^{\langle \ell \rangle} \cong \mathbb{L}_{\ell}(G^{\langle \ell \rangle}).$

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Uniqueness of original graphs

Xueliang Li proved that simple graphs of minimum degree \geqslant 3 are isomorphic if and only if their 2-path graphs are isomorphic. Li also conjectured that the assertion is true when minimum degree is 2, which was shown to be false by Aldred et al.

- $(\mathbb{L}_1(G))^{\langle \ell \rangle} \cong \mathbb{L}_{\ell}(G^{\langle \ell \rangle}).$
- For each ℓ ≥ 2, there are infinitely many pairs of simple graphs of minimum degree 2 with isomorphic ℓ-link (and ℓ-path) graphs.

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Uniqueness of original graphs

Let $\ell, s \geqslant 2$, and G and X be connected graphs of $\delta(G) \geqslant 3$ such that G is simple and $\mathbb{L}_{\ell}(G) \cong \mathbb{L}_{s}(X)$. In each of the following cases, $\ell = s$, $G \cong X$, $Aut(G) \cong Aut(\mathbb{L}_{\ell}(G))$, and every isomorphism from $\mathbb{L}_{\ell}(G)$ to $\mathbb{L}_{s}(X)$ is induced by an isomorphism from G to X:

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Uniqueness of original graphs

Let $\ell, s \geqslant 2$, and G and X be connected graphs of $\delta(G) \geqslant 3$ such that G is simple and $\mathbb{L}_{\ell}(G) \cong \mathbb{L}_{s}(X)$. In each of the following cases, $\ell = s$, $G \cong X$, $Aut(G) \cong Aut(\mathbb{L}_{\ell}(G))$, and every isomorphism from $\mathbb{L}_{\ell}(G)$ to $\mathbb{L}_{s}(X)$ is induced by an isomorphism from G to X:

• $\Delta(G) \geqslant 4$.

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Uniqueness of original graphs

Let $\ell, s \geqslant 2$, and G and X be connected graphs of $\delta(G) \geqslant 3$ such that G is simple and $\mathbb{L}_{\ell}(G) \cong \mathbb{L}_{s}(X)$. In each of the following cases, $\ell = s$, $G \cong X$, $Aut(G) \cong Aut(\mathbb{L}_{\ell}(G))$, and every isomorphism from $\mathbb{L}_{\ell}(G)$ to $\mathbb{L}_{s}(X)$ is induced by an isomorphism from G to X:

- $\Delta(G) \geqslant 4$.
- G contains a triangle.

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Uniqueness of original graphs

Let $\ell, s \geqslant 2$, and G and X be connected graphs of $\delta(G) \geqslant 3$ such that G is simple and $\mathbb{L}_{\ell}(G) \cong \mathbb{L}_{s}(X)$. In each of the following cases, $\ell = s$, $G \cong X$, $Aut(G) \cong Aut(\mathbb{L}_{\ell}(G))$, and every isomorphism from $\mathbb{L}_{\ell}(G)$ to $\mathbb{L}_{s}(X)$ is induced by an isomorphism from G to X:

- $\Delta(G) \geqslant 4$.
- G contains a triangle.
- girth(G) ≥ 5 and X contains an (s 1)-link whose end vertices are of degree at least 3.

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Recognition and determination algorithms

Roussopoulos (1973) gave an $\max\{m, n\}$ -time algorithm for determining the simple graph G from its line graph H.

 A graph is a 1-link graph if and only if it is the induced subgraph of the strong product of the line graph of a simple graph and some K_t².

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Recognition and determination algorithms

- A graph is a 1-link graph if and only if it is the induced subgraph of the strong product of the line graph of a simple graph and some K_t².
- It costs linear time O(m) to decide if H is a 1-link graph, and to find all its 1-roots.

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Recognition and determination algorithms

- A graph is a 1-link graph if and only if it is the induced subgraph of the strong product of the line graph of a simple graph and some K²_t.
- It costs linear time O(m) to decide if H is a 1-link graph, and to find all its 1-roots.
- For each finite graph H, it costs linear time O(m) to decide whether there exists an integer $\ell \geqslant 1$ and a graph G with $\delta(G) \geqslant 3$, such that $H \cong \mathbb{L}_{\ell}(G)$.

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Recognition and determination algorithms

- A graph is a 1-link graph if and only if it is the induced subgraph of the strong product of the line graph of a simple graph and some K²_t.
- It costs linear time O(m) to decide if H is a 1-link graph, and to find all its 1-roots.
- For each finite graph H, it costs linear time O(m) to decide whether there exists an integer $\ell \geqslant 1$ and a graph G with $\delta(G) \geqslant 3$, such that $H \cong \mathbb{L}_{\ell}(G)$.
- All such pairs (G, ℓ) can be obtained in linear time O(m).

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2-link graph

Let H be a 2-link graph. Then it has a vertex partition $\mathcal V$ and an edge partition $\mathcal E$ such that

Each part of V is an independent set of H;

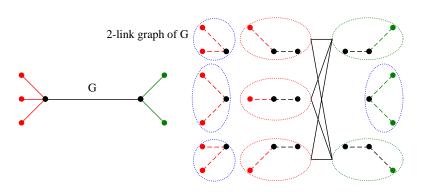


Figure: A 2-link graph of a simple graph is a 2-path graph

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2-link graph

Let H be a 2-link graph. Then it has a vertex partition $\mathcal V$ and an edge partition $\mathcal E$ such that

- Each part of \mathcal{V} is an independent set of H;
- Each part of \mathcal{E} induces a complete bipartite graph.

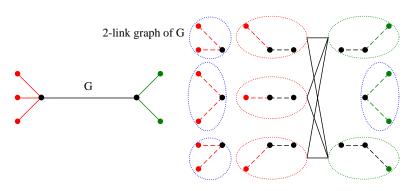


Figure: A 2-link graph of a simple graph is a 2-path graph

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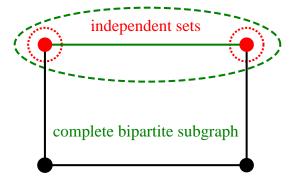
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Standard partition

 $(\mathcal{V}, \mathcal{E})$ is called a standard partition of H if further:



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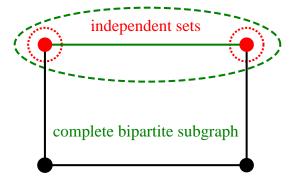
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Standard partition

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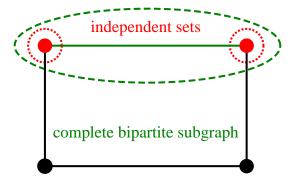
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Standard partition

 $(\mathcal{V}, \mathcal{E})$ is called a standard partition of H if further:

• Each part of $\mathcal E$ is incident to exactly two parts of $\mathcal V$.



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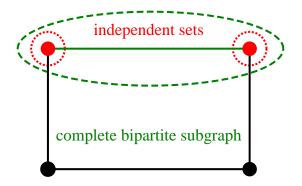
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Standard partition

 $(\mathcal{V}, \mathcal{E})$ is called a standard partition of H if further:

- Each part of $\mathcal E$ is incident to exactly two parts of $\mathcal V$.
- Each vertex of H is incident to exactly two parts of \mathcal{E} .



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Characterization of 2-link graphs

A graph is a 2-link graph of some graph of minimum degree at least 2 if and only if it admits a standard partition $(\mathcal{V}, \mathcal{E})$ such that:

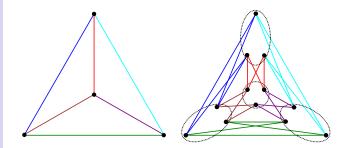


Figure: The 2-link graph of K_4

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Characterization of 2-link graphs

A graph is a 2-link graph of some graph of minimum degree at least 2 if and only if it admits a standard partition $(\mathcal{V}, \mathcal{E})$ such that:

 If E, F ∈ E are incident to some V ∈ V, then they are incident to exactly one vertex of V.

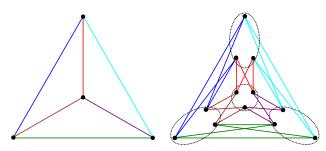


Figure: The 2-link graph of K_4

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Characterization of 2-link graphs

A graph is a 2-link graph of some graph of minimum degree at least 2 if and only if it admits a standard partition $(\mathcal{V}, \mathcal{E})$ such that:

 If E, F ∈ E are incident to some V ∈ V, then they are incident to exactly one vertex of V.

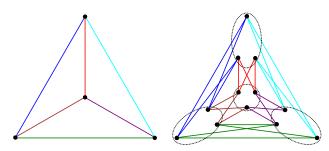


Figure: The 2-link graph of K_4

• $H \cong \mathbb{L}_2(G)$, where $G := H_{(\mathcal{V},\mathcal{E})}$.

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2-link graphs

Reduction of *ℓ*-links

• Let $\ell \geq 2$. Then an ℓ -link of G corresponds to a 2-link of the $(\ell - 2)$ -link graph of G.

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Reduction of ℓ -links

• Let $\ell \geq$ 2. Then an ℓ -link of G corresponds to a 2-link of the $(\ell-2)$ -link graph of G.



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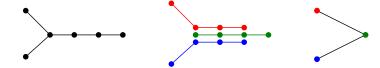
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Reduction of \(\ell \)-links

• Let $\ell \geq$ 2. Then an ℓ -link of G corresponds to a 2-link of the $(\ell-2)$ -link graph of G.



• But not vice versa.



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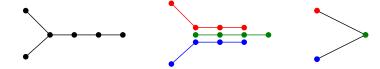
Ending

Reduction of *ℓ*-links

• Let $\ell \geq 2$. Then an ℓ -link of G corresponds to a 2-link of the $(\ell-2)$ -link graph of G.



• But not vice versa.



• $\mathbb{L}_{\ell}(G)$ is an induced subgraph of the 2-link graph of $\mathbb{L}_{\ell-2}(G)$.

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Characterization of 3-links graphs

Let H be a graph. Then H is a 3-link graph of some graph of minimum degree at least 2 if and only if there is a standard partition $(\mathcal{V},\mathcal{E})$ of H and a partition \mathcal{K} of \mathcal{E} such that:

• $H_{(\mathcal{V},\mathcal{E})}$ is a 1-link graph with an edge partition \mathcal{K} .

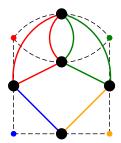


Figure: We only choose 2-links with two different colors

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Characterization of 3-links graphs

Let H be a graph. Then H is a 3-link graph of some graph of minimum degree at least 2 if and only if there is a standard partition $(\mathcal{V},\mathcal{E})$ of H and a partition \mathcal{K} of \mathcal{E} such that:

- $H_{(\mathcal{V},\mathcal{E})}$ is a 1-link graph with an edge partition \mathcal{K} .
- If $E, F \in \mathcal{E}$ are incident to $V \in \mathcal{V}$ in $H_{(\mathcal{V}, \mathcal{E})}$,

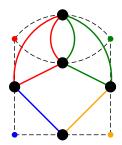


Figure: We only choose 2-links with two different colors

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Characterization of 3-links graphs

Let H be a graph. Then H is a 3-link graph of some graph of minimum degree at least 2 if and only if there is a standard partition $(\mathcal{V},\mathcal{E})$ of H and a partition \mathcal{K} of \mathcal{E} such that:

- $H_{(\mathcal{V},\mathcal{E})}$ is a 1-link graph with an edge partition \mathcal{K} .
- If $E, F \in \mathcal{E}$ are incident to $V \in \mathcal{V}$ in $H_{(\mathcal{V}, \mathcal{E})}$,
- then they are incident in H if and only if E and F are in different parts of K.

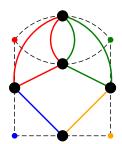


Figure: We only choose 2-links with two different colors

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Characterization of ℓ-link graphs

Let $\ell \geq 4$ be an integer. Then a graph H is an ℓ -link graph of a graph of minimum degree at least 2 if and only if H admits a standard partition $(\mathcal{V}, \mathcal{E})$ such that:

• $H_{(\mathcal{V},\mathcal{E})}$ is an $(\ell-2)$ -link graph of a graph of minimum degree at least 2, with a standard partition $(\mathcal{V}_{\ell-2},\mathcal{E}_{\ell-2})$.

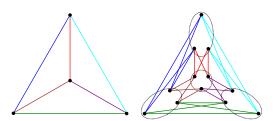


Figure: We only choose 2-links with two different colors

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Characterization of ℓ -link graphs

Let $\ell \geq 4$ be an integer. Then a graph H is an ℓ -link graph of a graph of minimum degree at least 2 if and only if H admits a standard partition $(\mathcal{V}, \mathcal{E})$ such that:

- $H_{(\mathcal{V},\mathcal{E})}$ is an $(\ell-2)$ -link graph of a graph of minimum degree at least 2, with a standard partition $(\mathcal{V}_{\ell-2},\mathcal{E}_{\ell-2})$.
- For any two different parts E, F of ε and any part V of V, E, F and V are incident at one vertex of H if and only if E_ε and F_ε are incident to V_ν, and correspond to two edges of H_(ν,ε) that are in different parts of ε_{ℓ-2}.

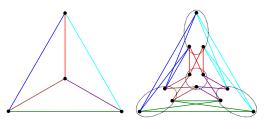


Figure: We only choose 2-links with two different colors

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Chromatic number

Recall that the chromatic number $\chi(G)$ of G is the smallest integer $t \geq 0$ such that V(G) can be colored by t colors and any two adjacent vertices are assigned to different colors.

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Coloring 2-link graphs

Let *H* be a 2-link graph of *G*. Then

• *H* is homomorphic to *G*.

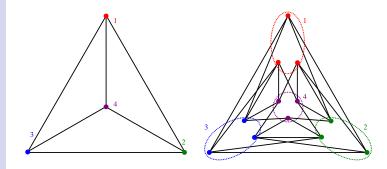


Figure: H inherits a coloring from G

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Coloring 2-link graphs

Let H be a 2-link graph of G. Then

- *H* is homomorphic to *G*.
- So H can be colored by $\{1, 2, \dots, \chi := \chi(G)\}$.

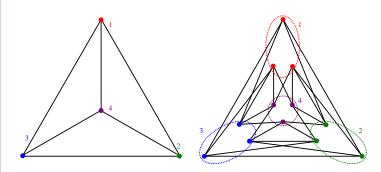


Figure: H inherits a coloring from G

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Every vertex of H is adjacent to at most two parts of V.

• For each $v \in V(H)$ colored by χ ,

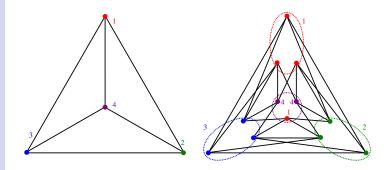


Figure: Change a color to the smallest possible integer

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Every vertex of H is adjacent to at most two parts of V.

- For each $v \in V(H)$ colored by χ ,
- the color of v can be replaced by one of $\{1,2,3\}$.

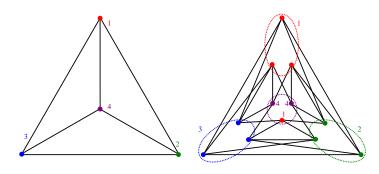


Figure: Change a color to the smallest possible integer

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H is homomorphic to *G*, so *H* can be colored by the color set $\{1, 2, ..., \chi(G)\}$. For each $v \in V(H)$ colored by χ , the color of v can be replaced by one of $\{1, 2, 3\}$.

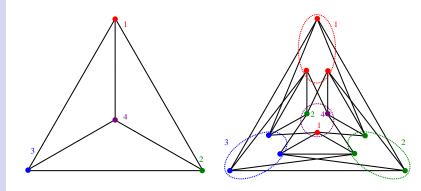


Figure: Change a color to the smallest possible integer

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Coloring

Let $\chi_{\ell} := \chi(\mathbb{L}_{\ell}(G))$.

• Repeat the process we obtain $\chi_2 \leq \lfloor \frac{2\chi}{3} \rfloor + 1$.

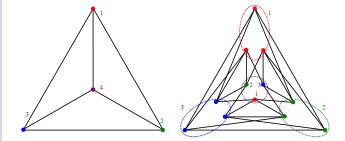


Figure: Change a color to the smallest possible integer

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Let $\chi_{\ell} := \chi(\mathbb{L}_{\ell}(G))$.

- Repeat the process we obtain $\chi_2 \leq \lfloor \frac{2\chi}{3} \rfloor + 1$.
- Note $\mathbb{L}_{\ell}(G)$ is a subgraph of the 2-link graph of $\mathbb{L}_{\ell-2}(G)$.

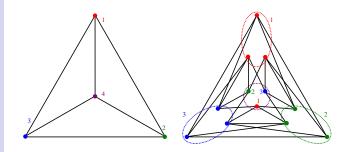


Figure: Change a color to the smallest possible integer

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Let $\chi_{\ell} := \chi(\mathbb{L}_{\ell}(G))$.

- Repeat the process we obtain $\chi_2 \leq \lfloor \frac{2\chi}{3} \rfloor + 1$.
- Note $\mathbb{L}_{\ell}(G)$ is a subgraph of the 2-link graph of $\mathbb{L}_{\ell-2}(G)$.
- So $\chi_{\ell} \leq \lfloor \frac{2\chi_{\ell-2}}{3} \rfloor + 1$.

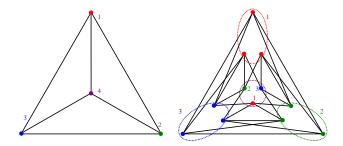


Figure: Change a color to the smallest possible integer

 ℓ -link graphs

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Let G be a graph with maximum degree Δ , chromatic number χ and edge chromatic number χ' . Let $\ell \geq 0$ be an integer and χ_{ℓ} be the chromatic number of the ℓ -link graph of G. Then

• If $\ell \geq 0$ is even, then $\chi_{\ell} \leq \min\{\chi, \lfloor (\frac{2}{3})^{\ell/2}(\chi - 3) \rfloor + 3\}$.

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- If $\ell \geq 0$ is even, then $\chi_{\ell} \leq \min\{\chi, \lfloor (\frac{2}{3})^{\ell/2}(\chi 3) \rfloor + 3\}$.
- If $\ell \geq 1$ is odd, then $\chi_{\ell} \leq \min\{\chi', \lfloor (\frac{2}{3})^{\frac{\ell-1}{2}}(\chi'-3)\rfloor + 3\}$.

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- If $\ell \geq 0$ is even, then $\chi_{\ell} \leq \min\{\chi, \lfloor (\frac{2}{3})^{\ell/2}(\chi 3) \rfloor + 3\}$.
- If $\ell \geq 1$ is odd, then $\chi_{\ell} \leq \min\{\chi', \lfloor (\frac{2}{3})^{\frac{\ell-1}{2}}(\chi'-3)\rfloor + 3\}$.
- If $\ell \geq 2$, then $\chi_{\ell} \leq \chi_{\ell-2}$.

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- If $\ell \geq 0$ is even, then $\chi_{\ell} \leq \min\{\chi, \lfloor (\frac{2}{3})^{\ell/2}(\chi 3) \rfloor + 3\}$.
- If $\ell \geq 1$ is odd, then $\chi_{\ell} \leq \min\{\chi', \lfloor (\frac{2}{3})^{\frac{\ell-1}{2}}(\chi'-3)\rfloor + 3\}$.
- If $\ell \geq 2$, then $\chi_{\ell} \leq \chi_{\ell-2}$.
- Recall that $\chi' \leq \frac{3}{2}\Delta$ (Shannon, 1949).
- If $\ell \neq 1$, then $\chi_{\ell} \leq \Delta + 1$.

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- If $\ell \geq 0$ is even, then $\chi_{\ell} \leq \min\{\chi, \lfloor (\frac{2}{3})^{\ell/2}(\chi 3) \rfloor + 3\}$.
- If $\ell \geq 1$ is odd, then $\chi_{\ell} \leq \min\{\chi', \lfloor (\frac{2}{3})^{\frac{\ell-1}{2}}(\chi'-3)\rfloor + 3\}$.
- If $\ell \geq 2$, then $\chi_{\ell} \leq \chi_{\ell-2}$.
- Recall that $\chi' \leq \frac{3}{2}\Delta$ (Shannon, 1949).
- If $\ell \neq 1$, then $\chi_{\ell} \leq \Delta + 1$.
- $\mathbb{L}_{\ell}(G)$ is tripartite for all $\ell > 5 \ln(\Delta 2) + 3$.

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- If $\ell \geq 0$ is even, then $\chi_{\ell} \leq \min\{\chi, \lfloor (\frac{2}{3})^{\ell/2}(\chi 3) \rfloor + 3\}$.
- If $\ell \geq 1$ is odd, then $\chi_{\ell} \leq \min\{\chi', \lfloor (\frac{2}{3})^{\frac{\ell-1}{2}}(\chi'-3)\rfloor + 3\}$.
- If $\ell \geq 2$, then $\chi_{\ell} \leq \chi_{\ell-2}$.
- Recall that $\chi' \leq \frac{3}{2}\Delta$ (Shannon, 1949).
- If $\ell \neq 1$, then $\chi_{\ell} \leq \Delta + 1$.
- $\mathbb{L}_{\ell}(G)$ is tripartite for all $\ell > 5 \ln(\Delta 2) + 3$.
- $\chi_{\ell} \le 6$ if $\ell > 5 \ln(\Delta 2) 3.8$.

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Edge Contraction

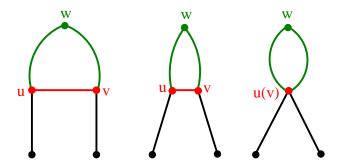


Figure: Contracting the edge uv

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Let G be a finite graph.

 A graph is said to be a minor of G if it can be obtained from a subgraph of G by contracting edges.

Graph minors

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Graph minors

Let *G* be a finite graph.

- A graph is said to be a minor of G if it can be obtained from a subgraph of G by contracting edges.
- The Hadwiger number $\eta(G)$ is the maximal number t such that G has K_t as a minor.

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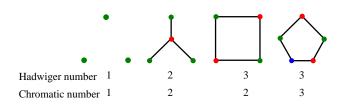
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Hadwiger and chromatic numbers



• In the above examples, we have $\eta(G) \ge \chi(G)$.

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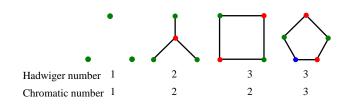
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Hadwiger and chromatic numbers



- In the above examples, we have $\eta(G) \ge \chi(G)$.
- Hadwiger's conjecture states that:

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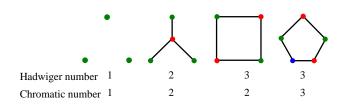
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Hadwiger and chromatic numbers



- In the above examples, we have $\eta(G) \ge \chi(G)$.
- Hadwiger's conjecture states that:
- For every finite graph G, $\eta(G) \ge \chi(G)$.

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Graph minors of link graphs

Let G be a graph of $\delta(G) \ge 2$, and X be a connected subgraph of G containing at least one ℓ -link. Then

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Graph minors of link graphs

Let *G* be a graph of $\delta(G) \geqslant 2$, and *X* be a connected subgraph of *G* containing at least one ℓ -link. Then

• For any two ℓ -links of X, one can be shunted to the other under the restriction that

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Graph minors of link graphs

Let *G* be a graph of $\delta(G) \geqslant 2$, and *X* be a connected subgraph of *G* containing at least one ℓ -link. Then

- For any two ℓ -links of X, one can be shunted to the other under the restriction that
- the middle vertices or edges of the images are inside X.

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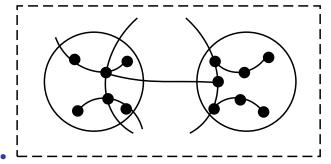
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Graph minors of link graphs

Let *G* be a graph of $\delta(G) \geqslant 2$, and *X* be a connected subgraph of *G* containing at least one ℓ -link. Then

- For any two ℓ -links of X, one can be shunted to the other under the restriction that
- the middle vertices or edges of the images are inside X.



 ℓ -link graphs

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Clique minors

If G contains an (ℓ, t) -system, then the ℓ -link graph of G contains a K_t -minor.

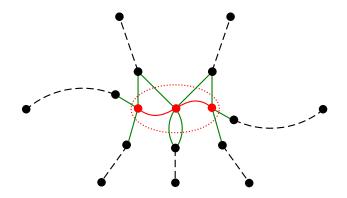


Figure: An $(\ell, 10)$ -system implies a K_{10} -minor

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Hadwiger's conjecture

 So far the best progress in Hadwiger's conjecture is achieved by Robertson, Seymour and Thomas in 1993.

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Hadwiger's conjecture

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Hadwiger's conjecture

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- They proved the conjecture is true for all graphs G of \(\chi(G) \le 6.\)
- In 2004, Reed and Seymour proved that the conjecture is true for all line graphs,

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Hadwiger's conjecture

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- They proved the conjecture is true for all graphs G of \(\chi(G) \le 6\).
- In 2004, Reed and Seymour proved that the conjecture is true for all line graphs,
- which is equivalent to say, Hadwiger's conjecture is true for all 1-link graphs.

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Our results

Hadwiger's conjecture for 0-link graphs is equivalent to the conjecture itself. We proved the conjecture for ℓ -link graphs of a graph G such that:

• $\ell > 5 \ln(\Delta(G) - 2) - 3.8$.

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Hadwiger's conjecture for 0-link graphs is equivalent to the conjecture itself. We proved the conjecture for ℓ -link graphs of a graph G such that:

- $\ell > 5 \ln(\Delta(G) 2) 3.8$.
- $\delta(G) \geq 3$ and $\ell \geq 1$.

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Hadwiger's conjecture for 0-link graphs is equivalent to the conjecture itself. We proved the conjecture for ℓ -link graphs of a graph G such that:

- $\ell > 5 \ln(\Delta(G) 2) 3.8$.
- $\delta(G) \ge 3$ and $\ell \ge 1$.
- $\ell \geq 2$ is an even integer.

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- $\ell \geq 2$ is an even integer.
- *G* is biconnected and $\ell \geq 1$.

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- $\delta(G) \ge 3$ and $\ell \ge 1$.
- $\ell \geq 2$ is an even integer.
- *G* is biconnected and $\ell \geq 1$.
- Another interesting result we obtained was that, if G contains a cycle, then η(L_ℓ(G)) ≥ η(G) for all ℓ ≥ 0.

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Minimal roots of cycles

Let $\mathbb{R}_{\ell}(H)$ be the set of ℓ -roots of H; that is, minimal graphs G such that $\mathbb{L}_{\ell}(G) \cong H$.

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Minimal roots of cycles

Let $\mathbb{R}_{\ell}(H)$ be the set of ℓ -roots of H; that is, minimal graphs G such that $\mathbb{L}_{\ell}(G) \cong H$.

• By Whitney, $\mathbb{R}_1(K_3) = \{K_3, K_{1,3}\}.$

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Minimal roots of cycles

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- $|\mathbb{R}_{\ell}(0)| \in \{1,2\}.$

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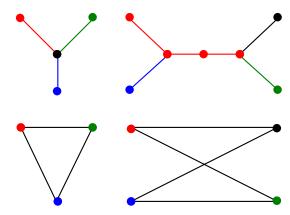
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Minimal roots of cycles

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- By Whitney, $\mathbb{R}_1(K_3) = \{K_3, K_{1,3}\}.$
- $|\mathbb{R}_{\ell}(0)| \in \{1,2\}.$
- Cycles with two minimal ℓ-roots



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• $\mathbb{R}_0(2K_1) = 2K_1$.

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- $\mathbb{R}_0(2K_1) = 2K_1$. $\mathbb{R}_1(2K_1) = 2K_2$.

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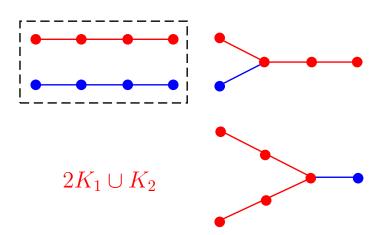
Minimal roots

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- $\mathbb{R}_0(2K_1) = 2K_1$.
- $\mathbb{R}_1(2K_1) = 2K_2$.
- $\mathbb{R}_2(2K_1) = 2P_2$.

Minimal roots

$$|\mathbb{R}_3(2K_1)|=2$$



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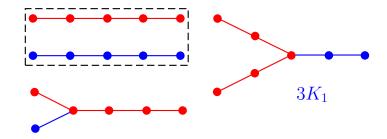
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$$|\mathbb{R}_4(2K_1)|=2$$



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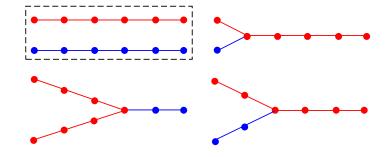
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Minimal roots

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 $|\mathbb{R}_5(2K_1)|=3.$ In general, $|\mathbb{R}_\ell(2K_1)|$ is 1 if $\ell=0,$ and is $|\frac{\ell+1}{2}|$ if $\ell\geqslant 1.$



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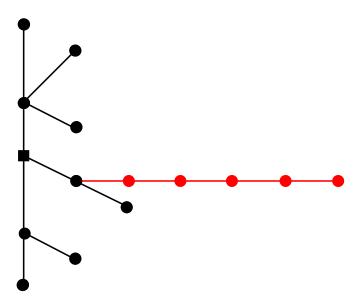
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Minimal 5-roots of a tree

The 5-link graph of the whole tree TP is the black subtree T.



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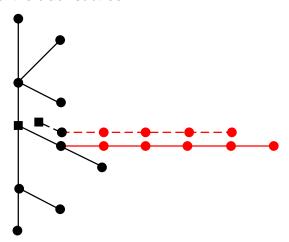
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Minimal 5-roots of a tree

 $\mathbb{L}_5(TP) \cong T$. Every 5-link has a unique black end; Every black node is the end of a unique 5-link. This gives a bijection between the 5-links of the whole tree TP and the nodes of the black subtree T.



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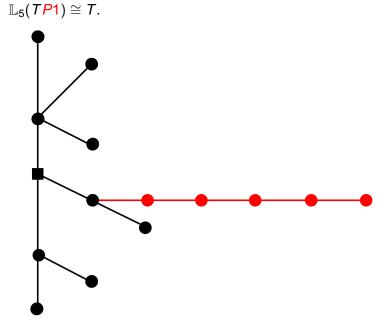
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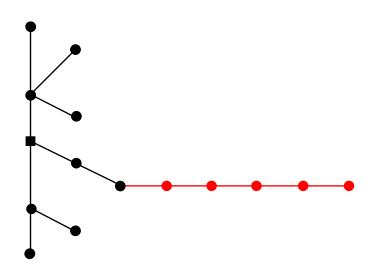


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 $\mathbb{L}_5(TP2) \cong T$.



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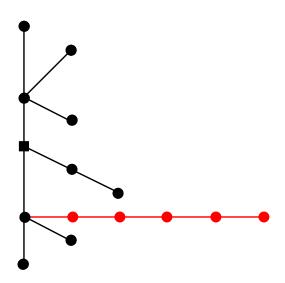
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$$\mathbb{L}_5(TP3) \cong T$$
.



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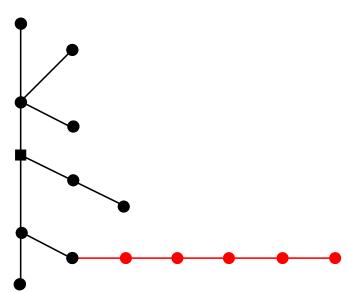
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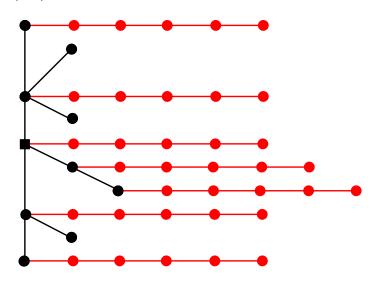
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 $\mathbb{L}_5(TP) \cong T$.





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Minimal ℓ -roots of a tree

For fixed $\ell \geqslant 4$ and any given number k, there exists a tree T with $|\mathbb{R}_{\ell}(T)| > k$.

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Bounding minimal roots

Let $\ell \geqslant 0$ be an integer, and H be a finite graph. Then the maximum degree, order, size, and total number of minimal ℓ -roots of H are finite and bounded by functions of H and ℓ .

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Bounding minimal roots

Let $\ell \geqslant 0$ be an integer, and H be a finite graph. Then the maximum degree, order, size, and total number of minimal ℓ -roots of H are finite and bounded by functions of H and ℓ .

 If the order and size are bounded, then other parameters above are trivially bounded.

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Bounding minimal roots

Let $\ell \geqslant 0$ be an integer, and H be a finite graph. Then the maximum degree, order, size, and total number of minimal ℓ -roots of H are finite and bounded by functions of H and ℓ .

- If the order and size are bounded, then other parameters above are trivially bounded.
- However, we improve the upper bounds by further investigating the structure of ℓ-link graphs.

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Bounding minimal roots

Let $\ell \geqslant 0$ be an integer, and H be a finite graph. Then the maximum degree, order, size, and total number of minimal ℓ -roots of H are finite and bounded by functions of H and ℓ .

- If the order and size are bounded, then other parameters above are trivially bounded.
- However, we improve the upper bounds by further investigating the structure of ℓ-link graphs.
- This is important in proving that the recognition problem belongs to \mathcal{NP} .

 ℓ -link graphs

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Minimal path roots

We say G is an ℓ -path root of H if $\mathbb{P}_{\ell}(G) \cong H$. Let $\mathbb{Q}_{\ell}(H)$ be the set of minimal ℓ -path roots of H.

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Minimal path roots

We say G is an ℓ -path root of H if $\mathbb{P}_{\ell}(G) \cong H$. Let $\mathbb{Q}_{\ell}(H)$ be the set of minimal ℓ -path roots of H.

 Xueliang Li (1996) proved that H has at most one simple 2-path root of minimum degree at least 3.

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Minimal path roots

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- Xueliang Li (1996) proved that H has at most one simple 2-path root of minimum degree at least 3.
- Prisner (2000) showed that $\mathbb{Q}_{\ell}(H)$ contains at most one simple graph of minimum degree greater than ℓ .

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Minimal path roots

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- Xueliang Li (1996) proved that H has at most one simple 2-path root of minimum degree at least 3.
- Prisner (2000) showed that $\mathbb{Q}_{\ell}(H)$ contains at most one simple graph of minimum degree greater than ℓ .
- Li and Yan Liu (2008) proved that, if H is connected and nonnull, then $\mathbb{Q}_2(H)$ contains at most two simple graphs.

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Minimal roots

Minimal path roots

We say G is an ℓ -path root of H if $\mathbb{P}_{\ell}(G) \cong H$. Let $\mathbb{Q}_{\ell}(H)$ be the set of minimal ℓ -path roots of H.

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- Li and Yan Liu (2008) proved that, if H is connected and nonnull, then $\mathbb{Q}_2(H)$ contains at most two simple graphs.
- The finite graphs having exactly two simple minimal 2-path roots have been characterised by Aldred, Ellingham, Hemminger and Jipsen (1997).

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Bounding minimal path roots

Let $\ell \geqslant 0$ be an integer, and H be a finite graph. We prove the following:

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Bounding minimal path roots

Let $\ell \geqslant 0$ be an integer, and H be a finite graph. We prove the following:

 The order, size, and total number of minimal ℓ-path roots of H are finite and bounded by functions of H and ℓ.

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Infiniteness of *ℓ*-roots

The maximum degree, order, size, and total number of ℓ -roots of a finite graph might be infinite.

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Infiniteness of *ℓ*-roots

The maximum degree, order, size, and total number of ℓ -roots of a finite graph might be infinite.

• The ℓ -roots of K_0 are trees of diameter at most $\ell - 1$.

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Infiniteness of *ℓ*-roots

The maximum degree, order, size, and total number of ℓ -roots of a finite graph might be infinite.

• The ℓ -roots of K_0 are trees of diameter at most $\ell - 1$.



Figure: All stars are 3-roots of K_0

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4-roots of K₁

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The maximum degree, order, size, and total number of ℓ -roots of a finite graph might be infinite.

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The maximum degree, order, size, and total number of ℓ -roots of a finite graph might be infinite.

• An ℓ -root of K_1 is a forest containing exactly one ℓ -path as a subgraph.

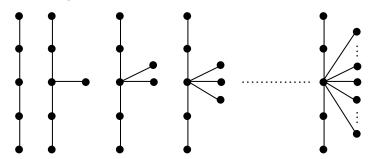


Figure: Connected 4-roots of K_1

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Constructing all ℓ-roots

Let G be the minimal graph of an ℓ -equivalence class. Then a graph belongs to this class if and only if it can be obtained from G as follows:

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Constructing all *ℓ*-roots

Let G be the minimal graph of an ℓ -equivalence class. Then a graph belongs to this class if and only if it can be obtained from G as follows:

• For each acyclic component T of G of diameter within $[\ell, 2\ell - 4]$, and every vertex u of eccentricity s in T such that $\lceil \ell/2 \rceil \leqslant s \leqslant \ell - 2$,

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Constructing all *ℓ***-roots**

Let G be the minimal graph of an ℓ -equivalence class. Then a graph belongs to this class if and only if it can be obtained from G as follows:

- For each acyclic component T of G of diameter within $[\ell, 2\ell 4]$, and every vertex u of eccentricity s in T such that $\lceil \ell/2 \rceil \leqslant s \leqslant \ell 2$,
- paste to u the root of a rooted tree of height at most $\ell s 1$.

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Constructing all *ℓ***-roots**

Let G be the minimal graph of an ℓ -equivalence class. Then a graph belongs to this class if and only if it can be obtained from G as follows:

- For each acyclic component T of G of diameter within $[\ell, 2\ell 4]$, and every vertex u of eccentricity s in T such that $\lceil \ell/2 \rceil \leqslant s \leqslant \ell 2$,
- paste to u the root of a rooted tree of height at most $\ell s 1$.
- Add to G zero or more acyclic components of diameter at most ℓ − 1.

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Constructing 4-roots of K_1

Every ℓ -root of a finite graph H can be constructed by a certain combination of a minimal ℓ -root of H and trees of bounded diameter.

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Constructing 4-roots of K_1

Every ℓ -root of a finite graph H can be constructed by a certain combination of a minimal ℓ -root of H and trees of bounded diameter.

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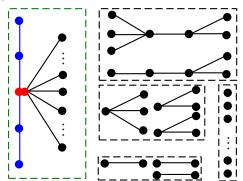


Figure: Constructing 4-roots of K_1 from a 4-path

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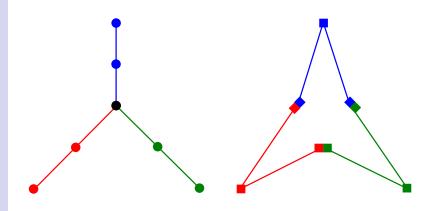
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ℓ -roots of a 3ℓ -cycles



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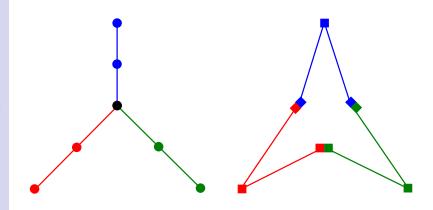
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ℓ-roots of a 3*ℓ*-cycles



A 6-cycle has two minimal 2-roots.

 ℓ -link graphs

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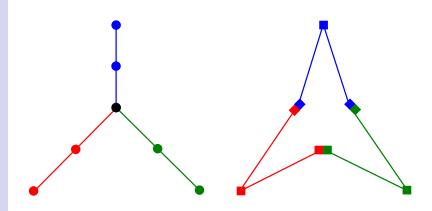
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ℓ -roots of a 3ℓ -cycles



- A 6-cycle has two minimal 2-roots.
- all 2-roots can be obtained by adding to one of them disjoint vertices or edges.

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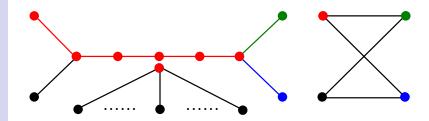
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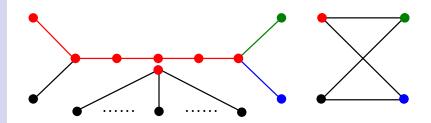
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A 4-cycle has two minimal 5-roots.

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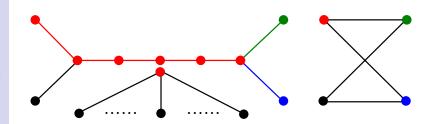
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- A 4-cycle has two minimal 5-roots.
- There are infinitely many trees of which the 5-link graph is a 4-cycle.

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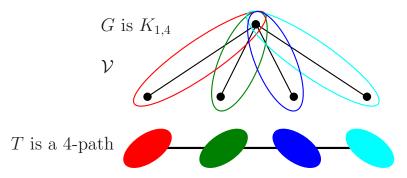
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Tree-decompositions

Let G be a graph, T be a tree, and $\mathcal{V} := \{V_w | w \in V(T)\}$ be a set cover of V(G) indexed by the nodes of T. The pair (T, \mathcal{V}) is called a *tree-decomposition* of G if . . .



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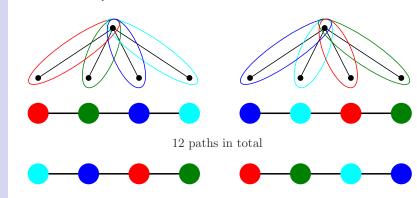
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Different indexes

Even if G, $\mathcal V$ and T are given, $\mathcal V$ can be indexed by V(T) in different ways.



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Different T

Even if G, V are given, there may be different T.

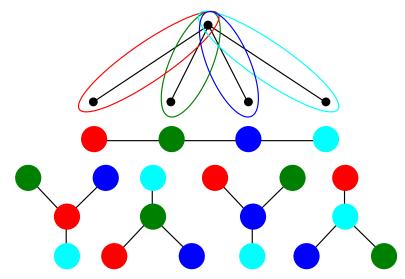


Figure: star- VS path-decomposition: diam(T)

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Tree-diameter of *ℓ*-roots

Let $\ell \geqslant 0$ be an integer, and H be a finite graph. Then

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Tree-diameter of ℓ-roots

Let $\ell \geqslant 0$ be an integer, and H be a finite graph. Then

 the tree-width and tree-diameter of the ℓ-roots and ℓ-path roots of H are finite and bounded by functions of H and ℓ.

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Quasi-ordering

A quasi-ordering is a binary relation \leqslant that is:

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Quasi-ordering

A quasi-ordering is a binary relation \leqslant that is:

• reflexive: $x \leqslant x$; and

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A quasi-ordering is a binary relation ≤ that is:

• reflexive: $x \le x$; and

• transitive: if $x \leqslant y$ and $y \leqslant z$, then $x \leqslant z$.

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Well-quasi-ordering

Let \leq be a quasi-ordering on X. X is said to be well-quasi-ordered if

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Well-quasi-ordering

Let \leq be a quasi-ordering on X. X is said to be well-quasi-ordered if

for any infinite sequence x₁, x₂,..., of X

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Well-quasi-ordering

Let \leq be a quasi-ordering on X. X is said to be well-quasi-ordered if

- for any infinite sequence x₁, x₂,..., of X
- there are indices i < j such that $x_i \leqslant x_j$.

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Well-quasi-ordering

Let \leq be a quasi-ordering on X. X is said to be well-quasi-ordered if

- for any infinite sequence x₁, x₂,..., of X
- there are indices i < j such that $x_i \leqslant x_j$.
- \bullet The indices for well-quasi-ordering are 1 < 2 < 3 $< \dots$

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Better-quasi-ordering: indices

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Better-quasi-ordering: indices

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• It includes the indices for well-quasi-ordering.

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Better-quasi-ordering: indices

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- It includes the indices for well-quasi-ordering.
- 1 ⊲ 2 ⊲ 4 ⊲

Better-quasi-ordering: indices

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- It includes the indices for well-quasi-ordering.
- 1 ⊲ 2 ⊲ 4 ⊲
- $(1,2,4,5) \triangleleft (2,4,5,7) \triangleleft (4,5,7,8,9)$.

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Better-quasi-ordering: blocks

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Better-quasi-ordering: blocks

A *block* \mathcal{B} is a set of finite increasing sequences B_1, B_2, \ldots such that:

• every infinite increasing sequence with elements in $B_1 \cup B_2 \cup \dots$

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Better-quasi-ordering: blocks

- every infinite increasing sequence with elements in $B_1 \cup B_2 \cup ...$
- has an initial finite subsequence that belongs to \mathcal{B} .

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Better-quasi-ordering: blocks

- every infinite increasing sequence with elements in $B_1 \cup B_2 \cup \dots$
- has an initial finite subsequence that belongs to \mathcal{B} .
- {1, 2, ...} is a block.

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Better-quasi-ordering: blocks

- every infinite increasing sequence with elements in $B_1 \cup B_2 \cup \dots$
- has an initial finite subsequence that belongs to \mathcal{B} .
- {1, 2, ...} is a block.
- For example, for each $n \ge 1$, the set of increasing sequence of $\mathbb N$ with n elements is a block.

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Better-quasi-ordering

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Let Q be a set quasi-ordered by \leq .

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Better-quasi-ordering

Let Q be a set quasi-ordered by \leq .

 A Q-pattern is a sequence of elements from Q that is indexed by a block B.

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Better-quasi-ordering

Let Q be a set quasi-ordered by \leq .

- A Q-pattern is a sequence of elements from Q that is indexed by a block B.
- A Q-pattern is good if there are $B \triangleleft C$ in \mathcal{B} , such that

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Better-quasi-ordering

Let Q be a set quasi-ordered by \leq .

- A Q-pattern is a sequence of elements from Q that is indexed by a block B.
- A Q-pattern is good if there are $B \triangleleft C$ in B, such that
- $q_B \leqslant q_C$. Otherwise, it is bad.

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Better-quasi-ordering

Let Q be a set quasi-ordered by \leq .

- A Q-pattern is a sequence of elements from Q that is indexed by a block B.
- A Q-pattern is good if there are $B \triangleleft C$ in \mathcal{B} , such that
- $q_B \leqslant q_C$. Otherwise, it is bad.
- Q is better-quasi-ordered if there is NO bad Q-pattern.

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Better-quasi-ordering: ℓ-roots

The ℓ -roots of a finite graph are better-quasi-ordered by the induced subgraph relation.

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Better-quasi-ordering: *ℓ*-roots

The ℓ -roots of a finite graph are better-quasi-ordered by the induced subgraph relation.

 The ℓ-path roots of a finite graph are better-quasi-ordered by the subgraph relation.

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Better-quasi-ordering: *ℓ*-roots

The ℓ -roots of a finite graph are better-quasi-ordered by the induced subgraph relation.

- The ℓ-path roots of a finite graph are better-quasi-ordered by the subgraph relation.
- The ℓ-path roots of bounded multiplicity of a finite graph are better-quasi-ordered by the induced subgraph relation.

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Minimal roots

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Better-quasi-ordering: ℓ-roots

The ℓ -roots of a finite graph are better-quasi-ordered by the induced subgraph relation.

- The ℓ-path roots of a finite graph are better-quasi-ordered by the subgraph relation.
- The ℓ-path roots of bounded multiplicity of a finite graph are better-quasi-ordered by the induced subgraph relation.
- Trees of bounded diameter are better-quasi-ordered by the induced subgraph relation.

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ℓ-path-free graphs

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ℓ -path-free graphs

Let H be a finite planar graph. Robertson and Seymour (1986, 1990) proved that finite H-minor-free graphs are well-quasi-ordered by the minor relation.

 Thomas (1989) generalised this result by showing that H-minor-free graphs are better-quasi-ordered by the minor relation.

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ℓ -path-free graphs

- Thomas (1989) generalised this result by showing that H-minor-free graphs are better-quasi-ordered by the minor relation.
- Ding (1992) proved that finite simple ℓ-path-free graphs are well-quasi-ordered by the induced subgraph relation.

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ℓ-path-free graphs

- Thomas (1989) generalised this result by showing that H-minor-free graphs are better-quasi-ordered by the minor relation.
- Ding (1992) proved that finite simple ℓ-path-free graphs are well-quasi-ordered by the induced subgraph relation.
- Another proof, based on tree-depth was given by Nešetřil and Ossona de Mendez (2012).

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ℓ-path-free graphs

- Thomas (1989) generalised this result by showing that H-minor-free graphs are better-quasi-ordered by the minor relation.
- Ding (1992) proved that finite simple ℓ-path-free graphs are well-quasi-ordered by the induced subgraph relation.
- Another proof, based on tree-depth was given by Nešetřil and Ossona de Mendez (2012).
- The H-minor-free graphs are better-quasi-ordered by the subgraph relation if and only if H is a disjoint union of paths.

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ℓ-path-free graphs

- Thomas (1989) generalised this result by showing that H-minor-free graphs are better-quasi-ordered by the minor relation.
- Ding (1992) proved that finite simple ℓ-path-free graphs are well-quasi-ordered by the induced subgraph relation
- Another proof, based on tree-depth was given by Nešetřil and Ossona de Mendez (2012).
- The H-minor-free graphs are better-quasi-ordered by the subgraph relation if and only if H is a disjoint union of paths.
- The *H*-minor-free graphs of bounded multiplicity are better-quasi-ordered by the induced subgraph relation if and only if *H* is a disjoint union of paths.

ℓ-link graphsJia and Wood

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Thanks

Thank You!

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Thanks

Thank You for listening!