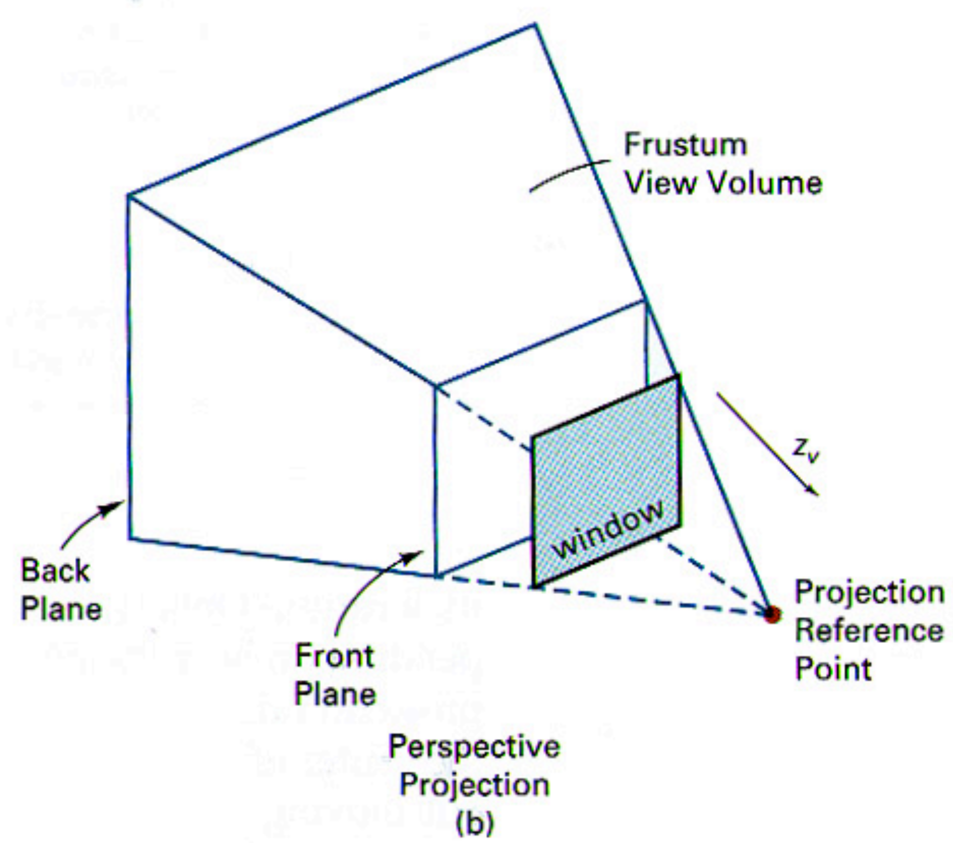
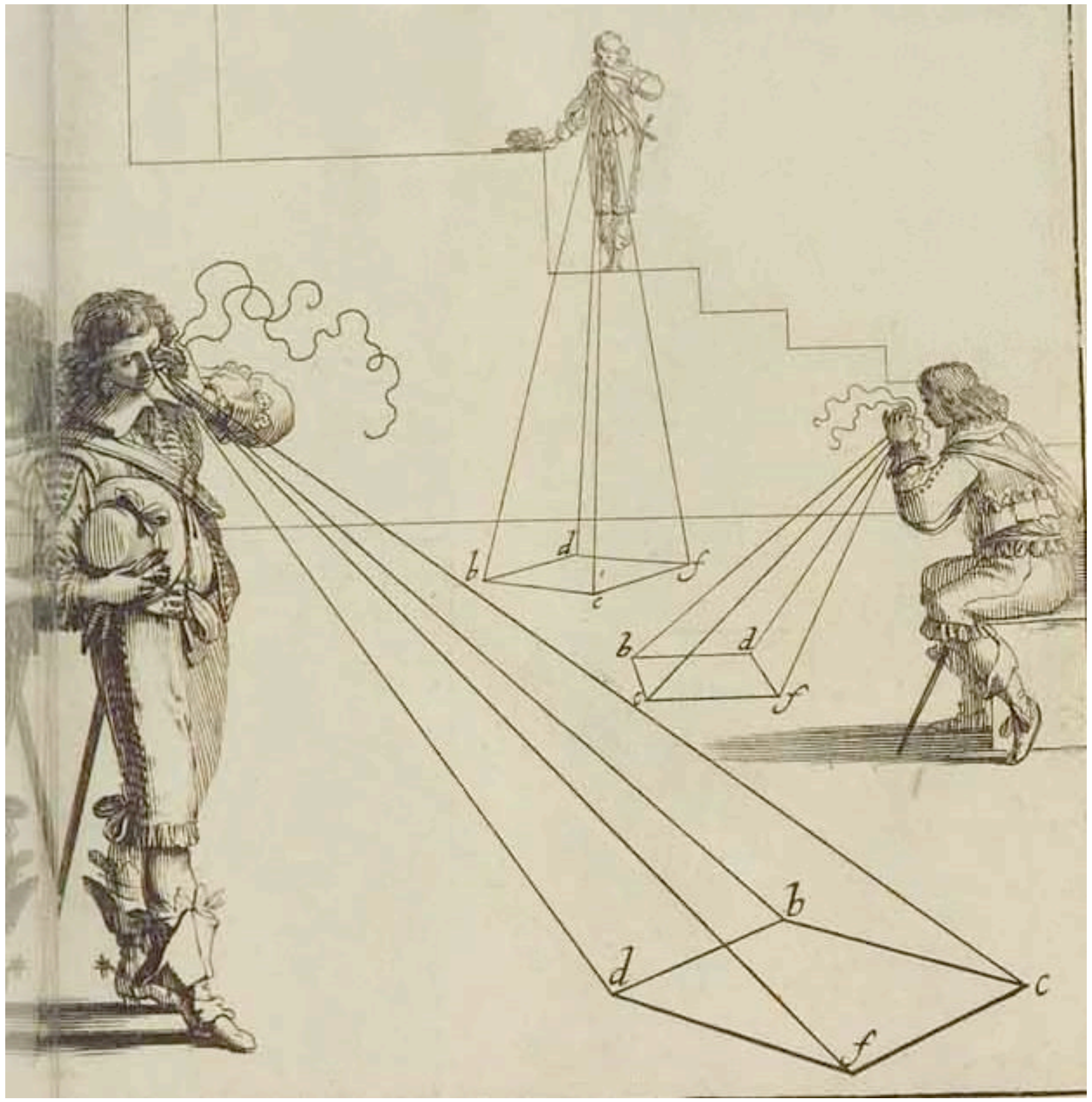
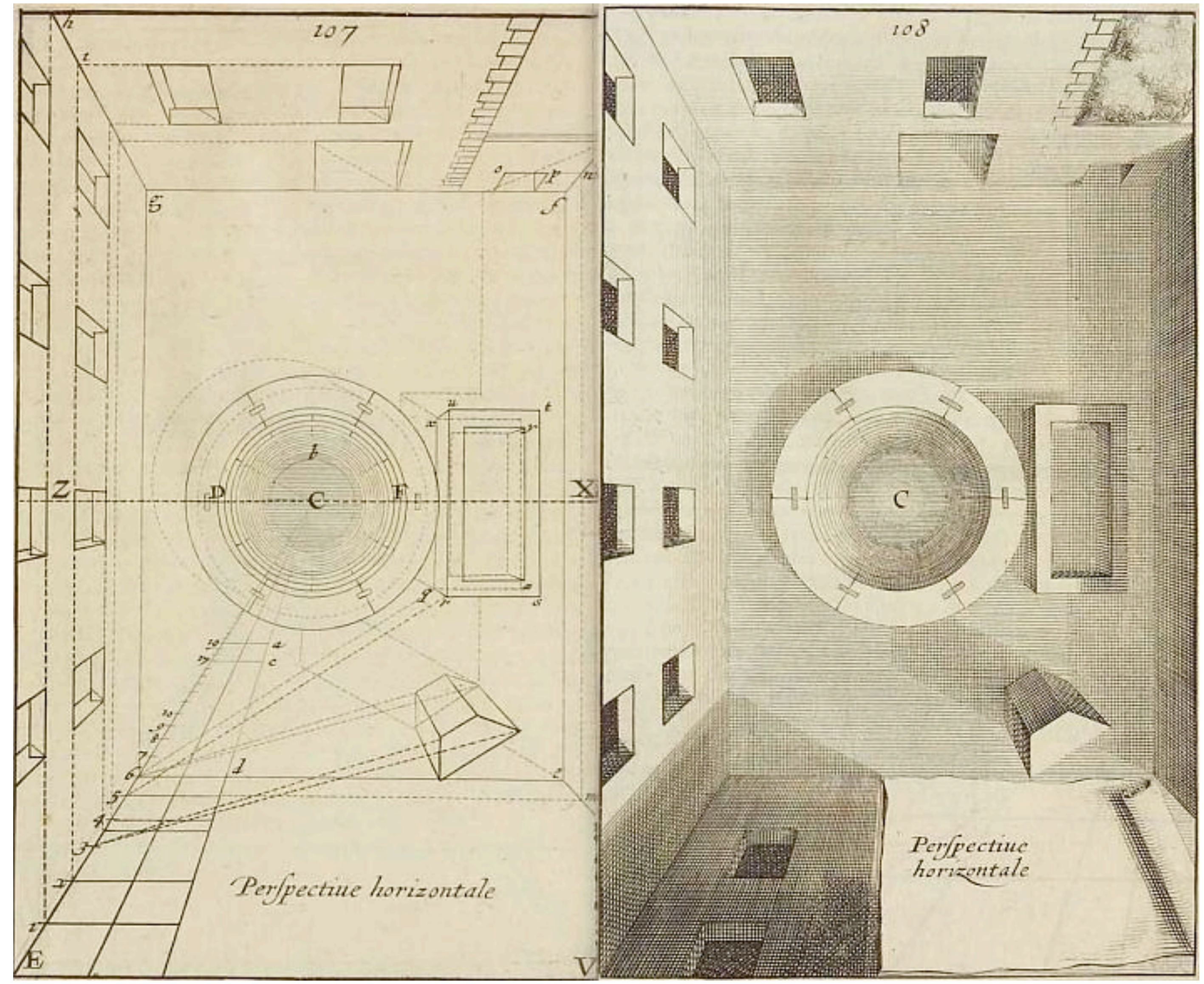


Monash University • Clayton's School of Information Technology

CSE3313 Computer Graphics

Lecture 17: Perspective Projection and Clipping





Manière Universelle de Desargues Pour Praticquer la Perspective par Petit Pied, Comme le Géométral (these images are from Volume I {Volume II})
by French mathematician Girard Desargues 1643-1647.
He is regarded as the founder of projective or perspective geometry.

Perspective Projection (cont.)

- To obtain a perspective projection of a 3D object, we project points along projection lines which meet at the *centre of projection*.
- For convenience, we can transform the coordinate system so that the centre of projection lies at $(0, 0, -d)$ of a left-handed coordinate system where the projection plane is the $z = 0$ plane.
- The projection line connecting any point (x, y, z) to the centre of projection $(0, 0, -d)$ can be expressed in parametric form as:

  $$x' = x - xt$$

  $$y' = y - yt$$

  $$z' = z - (z + d)\, t$$

  where the parameter t takes the values from 0 to 1.
- To obtain the coordinates on the projection plane the value of t for which $z = 0$.

Perspective Projection (cont.)

- Thus:

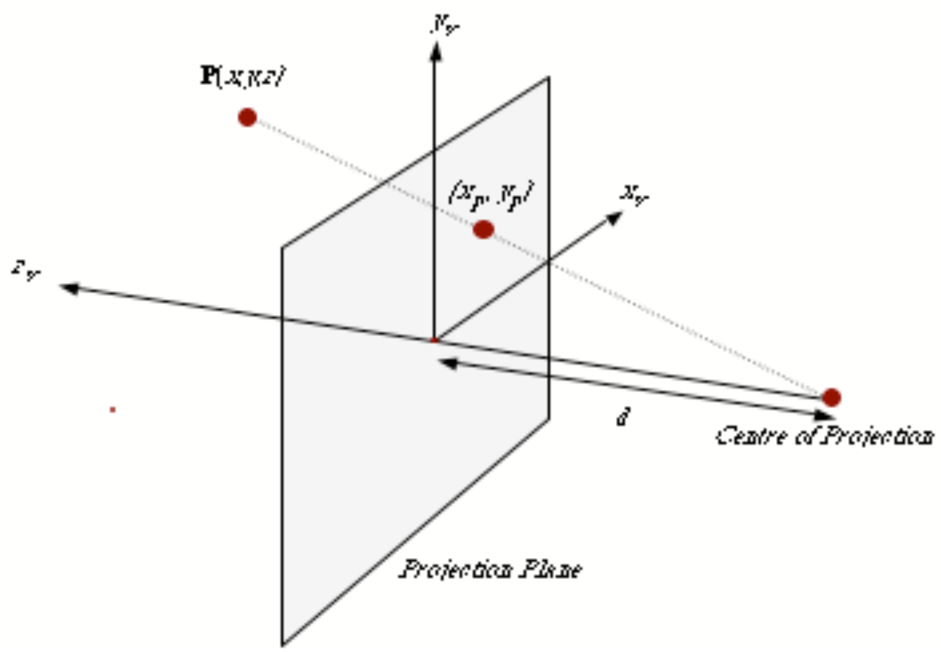
  $$t = z/(z+d)$$

- Substituting this value of t into the parametric form we get the x and y coordinates for the intersection with the view plane.

$$x_p = x\left(\frac{d}{z+d}\right) = x\left(\frac{1}{z/d+1}\right)$$

$$y_p = y\left(\frac{d}{z+d}\right) = y\left(\frac{1}{z/d+1}\right)$$

$$z_p = 0$$



Perspective Projection (cont.)

- Using homogenous coordinates the perspective transformation can be written in matrix form as:

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix}$$

In this representation: $w = \frac{z}{d} + 1 = \left(\frac{d}{z+d} \right)^{-1}$

If we consider the limit as $d \rightarrow \infty$, the perspective transformation tends to a parallel projection.

i.e. the parallel projection might be considered a perspective projection with the centre of projection at $(0, 0, -\infty)$.

View Volumes

- A rectangular window on the view plane, the **projection window**, determines what features projected onto the view plane will be visible. This window is defined for minimum and maximum values for x and y on the view plane.

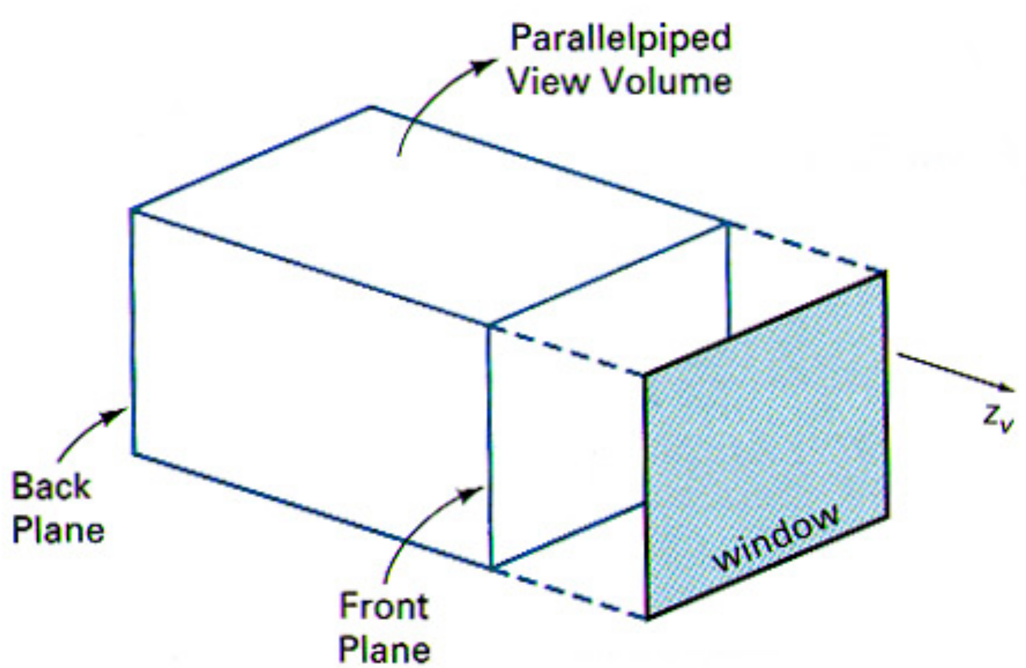
- The projection window is used to define a **view volume**. Only objects within the view volume are projected and displayed on the view plane.

- The shape of the view volume depends on the kind of projection used:

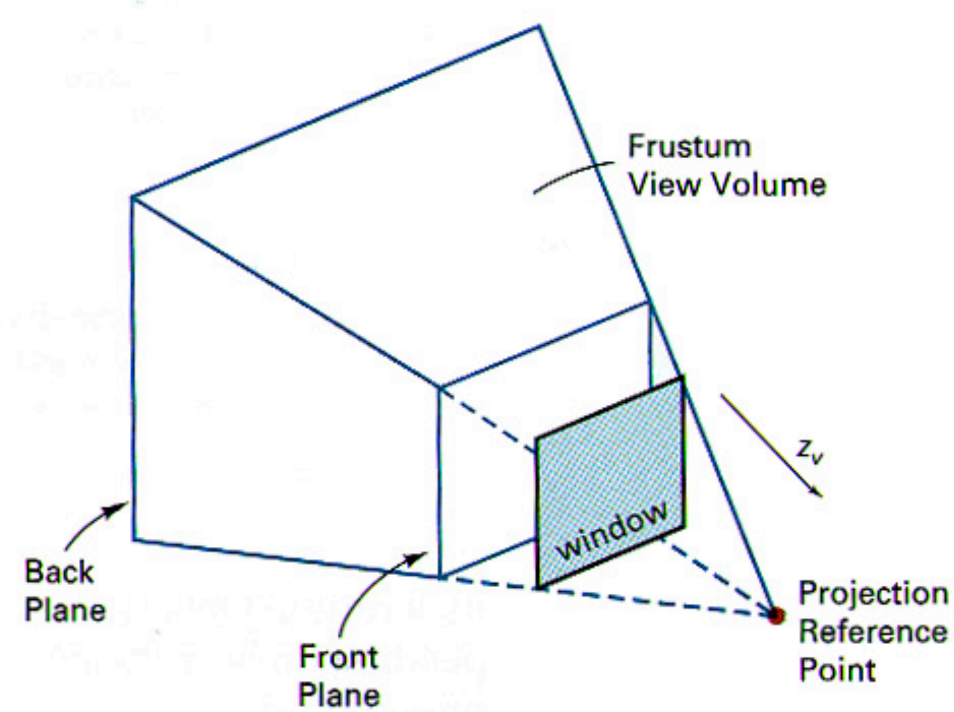
  - parallel projection: an infinite parallelepiped with the four sides of the view volume passing through the edges of the projection window.
  - perspective projection: a truncated pyramid, called a frustum, with the apex at the centre of projection and the four sides of the projection window forming the base of the pyramid.

View Volumes (cont.)

- Additional clipping planes may be introduced to reduce the infinite extent of the view volume.
- Objects that lie behind the **front** or hither clipping plane, or those that lie beyond the **back** or yon clipping plane, will not be visible.
- These planes are also referred to as the **near** and **far** clipping planes.



Parallel Projection
(a)



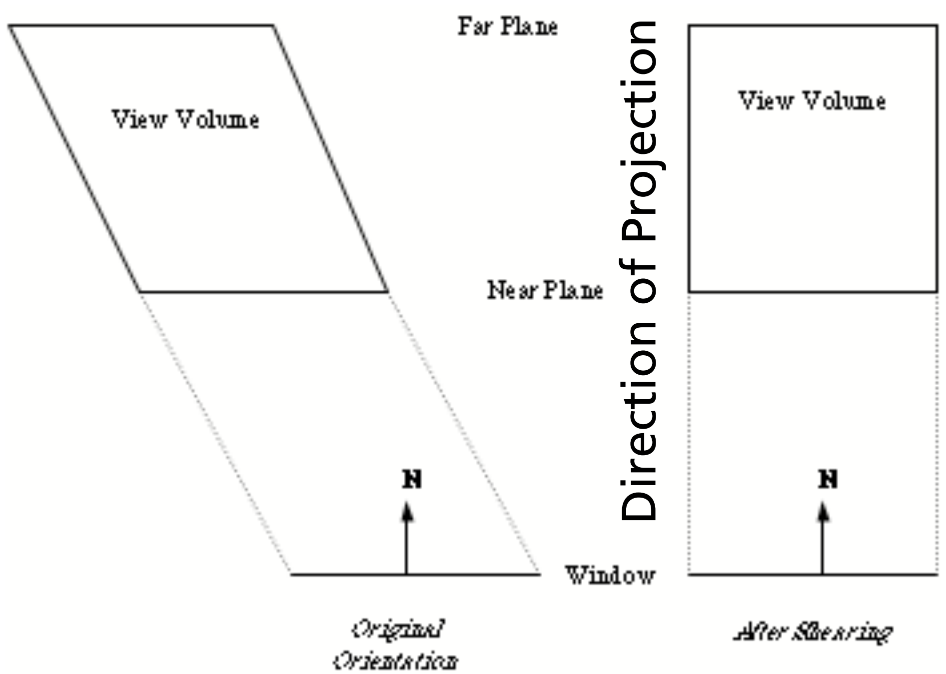
Perspective
Projection
(b)

Clipping

- 3D clipping must identify and save all points within the view volume.
- 2D clipping techniques can be extended to test line endpoints relative to each boundary of the view volume and calculate intersections where necessary.
- All clipping could be carried out in the 3D view volume or clipping can be carried out in the 2D projection plane using the same techniques as in 2D clipping.
- For this second approach to work, clipping against the front and back planes must have first been done in 3D coordinates.
- Clipping against the front and back planes is straightforward since both these planes are planes of constant z.

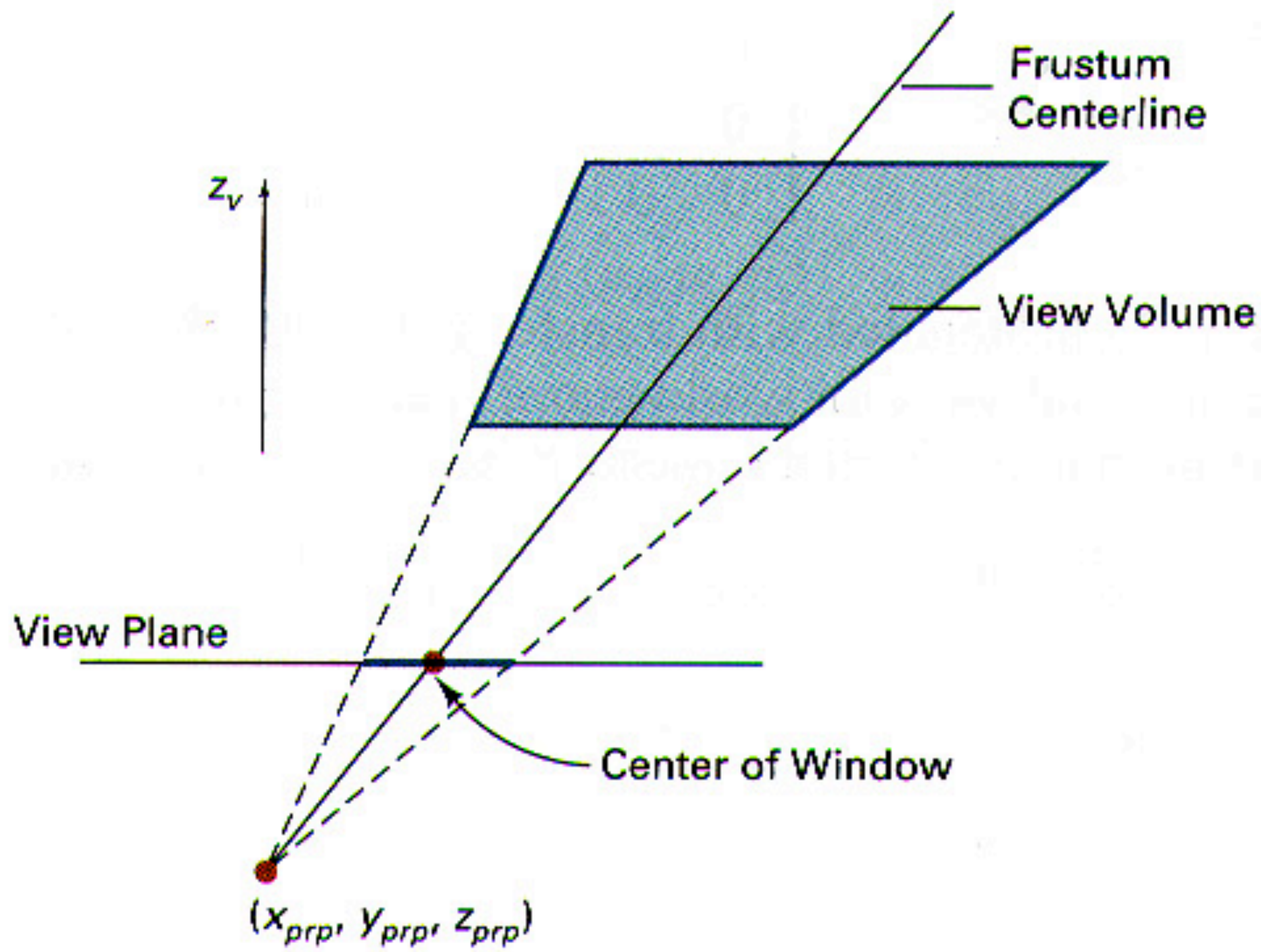
Clipping (cont.)

- Clipping against the sides of the view volume involves computing intersections with these planes. To do this we need to find equations for those planes.
- This can be avoided if we transform the view volume into a standardised 3D space. This standard volume is usually a rectangular parallelepiped.
- Each surface is parallel with the coordinate axis. Converting to a rectangular parallelepiped reduces the projection process to a simple orthogonal projection.



Clipping (cont.)

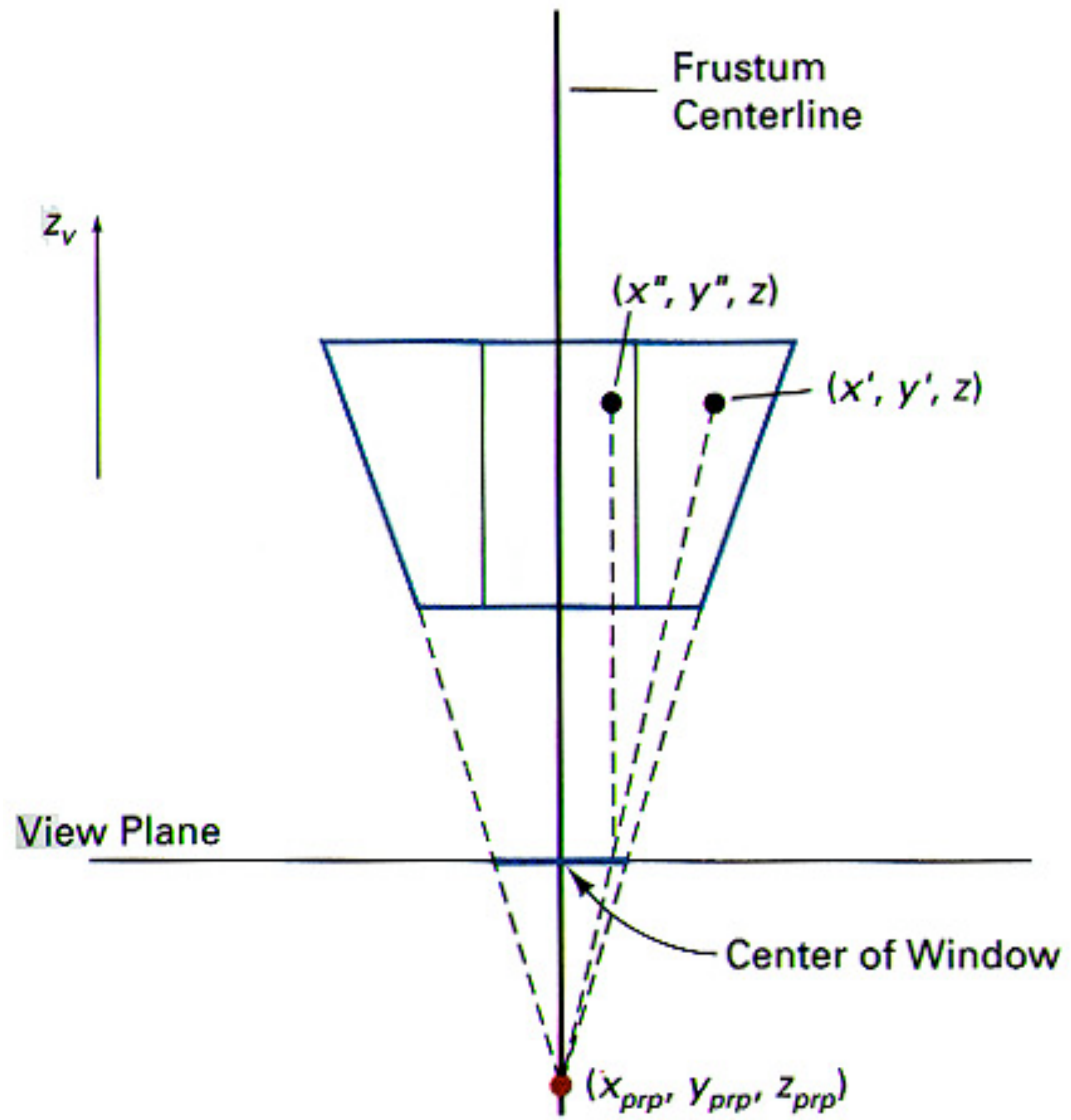
- For perspective projections we need to transform so that the centre of projection is on a line normal to the centre of the window. This is accomplished using a shear transformation.



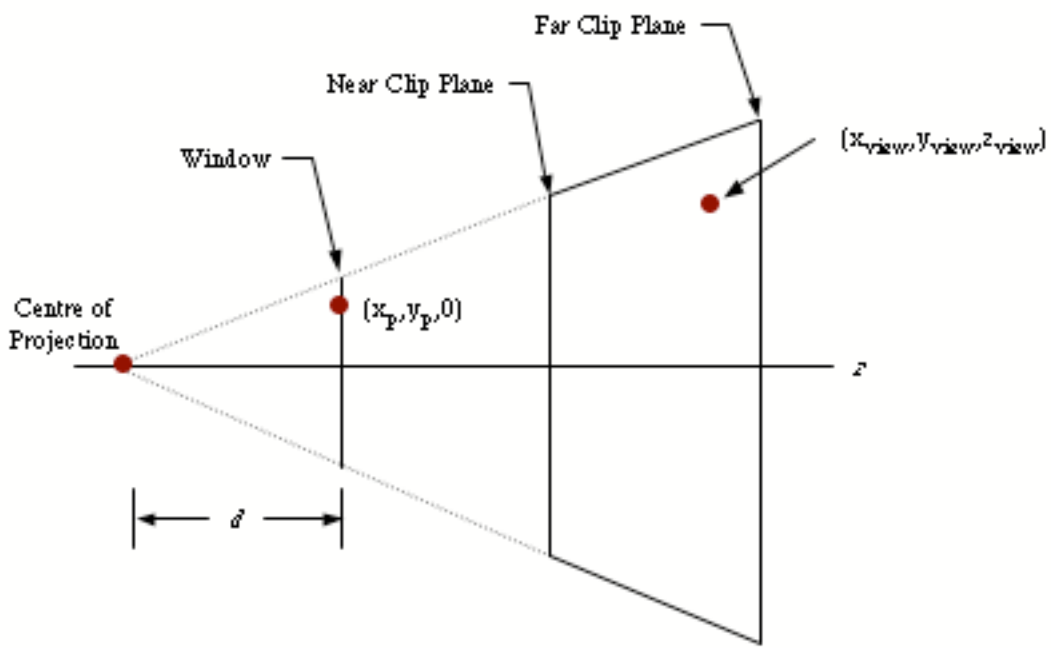
General shape for the perspective view volume with a projection reference point that is not on the z_v axis.

Clipping (cont.)

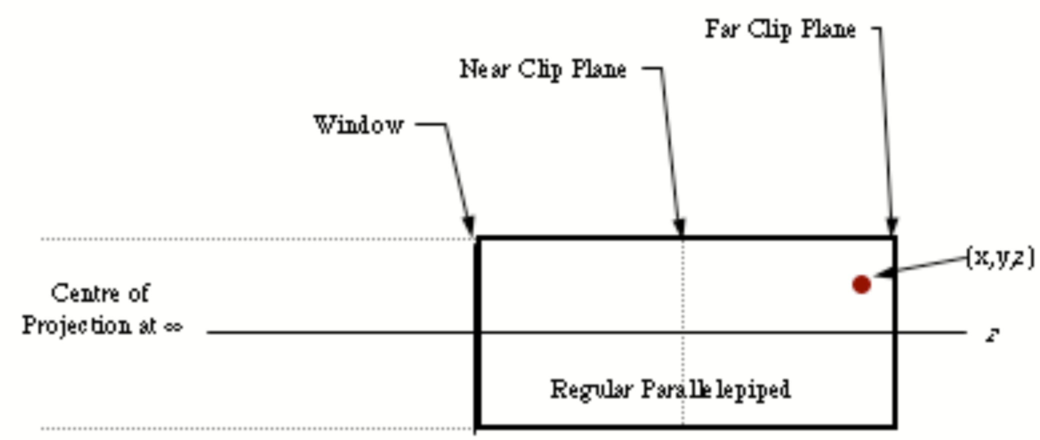
- We then scale the sides of the frustum so they form the regular sides of a parallelepiped.



- This last transformation has essentially done the work of the perspective transformation since the x and y values of a point in the view volume are also the same for the projection of the point onto the view plane.



Before Transformation



After Transformation

Perspective Transformation (cont.)

- The z coordinates are still important because they may be needed for hidden surface elimination.

- If two points project to the same coordinates on the plane, then the point with the smaller z coordinate will be visible as it is closer to the viewer that the point with the larger z coordinate, and both points are on the same "line of sight".

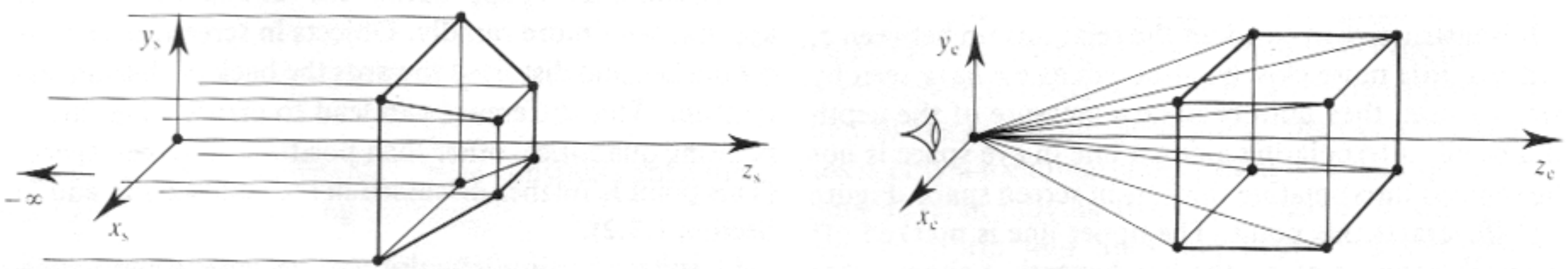


Figure 1.12 Transformation of box and light rays from eye space to screen space.